DESIGN RELIABILITY CONSIDERING FLUCTUATING HARMONIC LOAD

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ABSTRACT

The following is a method of calculating the fatigue strength of mechanical elements. The mentioned method, convenient for designers, is based on the systematic analysis of experimental data from different sources.

INTRODUCTION

Fatigue propagation, reliability, and safety is the focus of many researchers and designers. Since its discovery in the middle of the last century, the fatigue caused by fluctuating loads has become a gradually dominant mechanism of the majority of structures and, as the statistics on service failures show, also the dominant criterion of service reliability. This is related mainly to a constant pressure to economically use the material with the increasing service loads and decreasing safety factors. Many methods exist to design parts for fatigue; however, experiments show that it is not possible to conclude that one of these methods would be significantly comprehensive or give trustworthy estimates of service life. Of course, their complications and number of parameters needed must also be considered.

FLUCTUATING HARMONIC LOAD

Fluctuating stress (Fig. 1) can be determined by two arbitrary values from the following five: σ_{max}, σ_m, σ_{min}, σ_a, and R.

The relationship between these terms is given by three equations:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad R = \frac{\sigma_{min}}{\sigma_{max}}$$

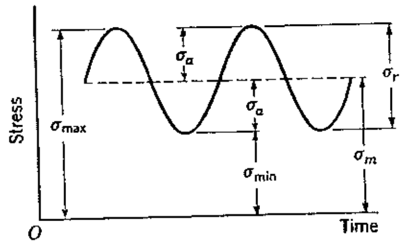

Fig. 1. Fluctuating Harmonic Stress

Experiments demonstrate that the fatigue strength is different in axial load (tension - compression) or bending or torsion.

DESIGN CRITERIA FOR A SINGLE CYCLIC LOAD

Reversed Cyclid Load

In the case of reversed cyclic stress (R = -1), the endurance limit of a polished specimen of low carbon steel with a 10 mm diameter can be expressed with the precision ± 10% by the following equations (Silovsky, Oliva 1959 & 1978):

in bending

$$S'_{be} = 0.25\ (S_{yt} + S_{ut}) + 50\ [\text{MPa}] \quad (1a)$$

in axial load (tension-compression)

$$S'_{te} = 0.8\ S'_{be} \quad (1b)$$

in torsion (shear)

$$S'_{se} = 0.6\ S'_{be} \quad (1c)$$

The endurance limit of an actual part is usually obtained by multiplying the endurance limit of the specimen by modifying factors (Shigley-Mitchell 1983):

$$S_e = k_a\ k_b\ k_c\ k_e\ S'_e \quad (2)$$

where $k_i \le 1$ represents the following modifying factors:

k_a - surface finish factor

k_b - size factor

k_c - reliability factor

k_e - stress concentration factor

By multiplying the modifying factors k_a and k_e, their common effect is over-estimated. Therefore, a more realistic result is obtained if, for the resultant stress concentration factor is used the relation (Tauscher 1956, Puchner 1959, Silovsky-Oliva 1959, Serensen-Kogayev-Schneiderovitch 1975):

$$K_r = 1 + \sum_{i=1}^{n} (K_i - 1) \qquad \text{where } K_i = \frac{1}{k_i} \ge 1. \quad (3)$$

It has been proven by experiments that for torsion (shear), the surface finish factor is smaller than for tension or bending. This value can be estimated using the relation (Silovsky-Oliva 1959, Puchner-Nemec 1971):

$$K_{sa} = 0.5 + 0.5\ K_a \qquad \text{or} \qquad k_{sa} = \frac{2k_a}{k_a + 1} \quad (4)$$

Then the endurance limit of an actual part is:

in bending

$$S_{be} = k_b k_c k_r S'_{be} \ \text{for}\ k_r = \frac{1}{K_r} \le 1 \quad \text{or} \quad S_{be} = \frac{k_b k_c}{K_r} S'_{be} \ \text{for}\ K_r = \frac{1}{k_r} \ge 1 \quad (5a)$$

in axial load (tension-compression)

$$S_{te} = k_c k_r S'_{te} \quad \text{or} \quad S_{te} = \frac{k_c}{K_r} S'_{te} \quad (5b)$$

in torsion

$$S_{se} = k_b k_c k_{sr} S'_{se} \quad \text{or} \quad S_{se} = \frac{k_b k_c}{K_{sr}} S'_{se} \quad (5c)$$

The safety factor of a specimen in reversed cyclic load (R = -1) with the amplitude σ_a s given by

$$n = \frac{S'_e}{\sigma_a}$$

and for shear stress

$$n = \frac{S'_{se}}{\tau_a}$$

The safety factor of an actual part is then

$$n = \frac{S_e}{\sigma_a} \qquad \text{or} \qquad n = \frac{S_{se}}{\tau_a} \quad (6)$$

Fluctuating Cyclic Load

Fluctuating stress cycles may be described using the working (e.g. Smith's or Haigh's) fatigue diagrams (Fig. 2).

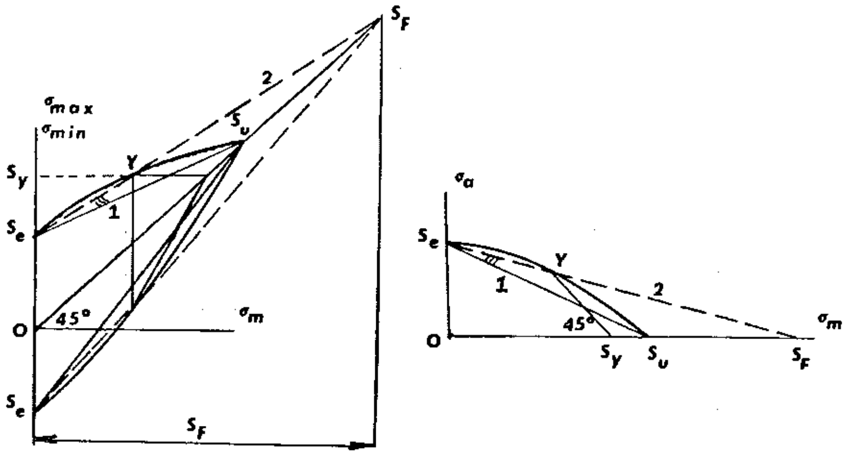

Fig. 2. Working Fatigue Diagrams

The limiting line represents the fatigue strength of any fluctuating cyclic load described by the amplitude and mean stress. For practical use by designers, these lines are simplified so that the curves are replaced by straight lines. This line is possible to plot in different modes, but the best fit is to plot the line through the endurance limit S_e (or S_e') of the reversed cycle ($R = -1$) and through "ficticious" ultimate strength S_F, which can be approximately determined using the following equations:

in axial load

$$S_{tF} = 3.5\ S_{te}' + 400\ [\text{MPa}] \tag{7a}$$

in bending

$$S_{bF} = 3.5\ S_{be}' + 400\ [\text{MPa}] \tag{7b}$$

in torsion

$$S_{sF} = 6\ S_{se}' \tag{7c}$$

When the given cycles have σ_m and σ_a, the common assumption for determining

fatigue strength is that both components are increasing in the same ratio. Because

$$\frac{\sigma_a}{\sigma_m} = \frac{\frac{\sigma_{max} - \sigma_{min}}{2}}{\frac{\sigma_{max} + \sigma_{min}}{2}} = \frac{1 - \frac{\sigma_{min}}{\sigma_{max}}}{1 + \frac{\sigma_{min}}{\sigma_{max}}} = \frac{1-R}{1+R}$$

it is possible that all cycles with the same ratio $\frac{\sigma_{min}}{\sigma_{max}}$ (=R) can be interpreted as a straight line OD (Fig. 3).

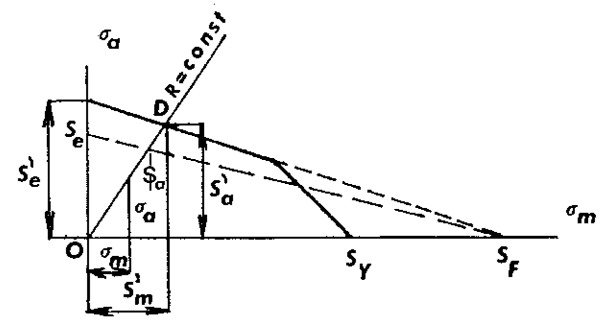

Fig. 3. Stress Cycles with Equal R

The strength values S_a', S_m' are then the coordinates of the interception point D. The safety factor is then given by the relation:

for specimen

$$n = \frac{S_a'}{\sigma_a} = \frac{S_m'}{\sigma_m} \tag{8a}$$

for actual part

$$n = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} \tag{8b}$$

Analytically, the strength values are determined from equations:

for specimen

$$S_a' = \frac{S_F}{\frac{\sigma_m}{\sigma_a} + \frac{S_F}{S_e'}} \qquad S_m' = \frac{S_e'}{\frac{\sigma_a}{\sigma_m} + \frac{S_e'}{S_F}} \tag{9}$$

for actual part

$$S_a = \frac{S_F}{\frac{\sigma_m}{\sigma_a} + \frac{S_F}{S_e}} \qquad S_m = \frac{S_e}{\frac{\sigma_a}{\sigma_m} + \frac{S_e}{S_F}} \tag{10}$$

It is also necessary to compute the safety factor against yielding

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_m + \sigma_a} \qquad (11)$$

Note: For torsion (special case of plane stress state) in equations (8,9,10,11) the values τ_m, τ_a, S_{sm}, S_{sy}, S_{se}, S_{sF} are used.

DESIGN CRITERIA FOR A COMBINED CYCLIC LOAD

Assuming each of the loads to be combined is in the same phase, the resultant stress for estimating the fatigue strength can be obtained either by algebraic summation (if the stresses are the same type and direction - e.g., the normal tensile and normal bending stresses) or using failure theories (if the stresses are of different type - e.g., the normal bending stress and torsional shear stress) or by geometrical summation (e.g., the stresses of the same type, but different direction, like in biaxial traction). Of the known theories of failure, only the theory of distortion energy (Huber-Mises-Hencky) can be used.

Combined Fluctuating Load with Equal Coefficients of Nonsymmetry

The simplest case of fluctuating load of multiaxial stress states occurs when the coefficients of nonsymmetry of stress cycles are equal

$$\left(R_1 = \frac{\sigma_{min}}{\sigma_{max}} = R_2 = \frac{\sigma_{2min}}{\sigma_{2max}} = R_3 = \frac{\sigma_{3min}}{\sigma_{3max}}\right)$$

Reversed cyclic load

If $R_1 = R_2 = R_3 = -1$ the safety factor of a specimen

$$n = \frac{S'_{te}}{\sigma_{ea}} \quad \text{or} \quad n = \frac{S'_{be}}{\sigma_{ea}} \qquad (12)$$

Fluctuating cyclic load

The safety factor of a speciman

$$n = \frac{S'_{ta}}{\sigma_{ea}} \quad \text{or} \quad n = \frac{S'_{ba}}{\sigma_{ea}}$$

eventually

$$n = \frac{S'_{tm}}{\sigma_{em}} \quad \text{or} \quad n = \frac{S'_{bm}}{\sigma_{em}} \qquad (13)$$

The equivalent amplitude

$$\sigma_{ea} = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 + \sigma_{3a}^2 - \sigma_{1a}\,\sigma_{2a} - \sigma_{2a}\,\sigma_{3a} - \sigma_{3a}\,\sigma_{1a}} \qquad (14)$$

and σ_{1a}, σ_{2a}, σ_{3a} in Equations 12 and 13 are amplitudes of fluctuating principal stresses σ_1, σ_2, and σ_3.

Similarly,

$$\sigma_{em} = \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 + \sigma_{3m}^2 - \sigma_{1m}\,\sigma_{2m} - \sigma_{2m}\,\sigma_{3m} - \sigma_{3m}\,\sigma_{1m}} \qquad (15)$$

The value S'_a (S'_{ta} or S'_{ba}) is the limiting amplitude of a specimen (under tension - compression, or bending) determined (Fig. 4) for given R from fatigue diagram (in tension - compression or bending) or analytically from the expression

$$S'_a = \frac{S_F}{\dfrac{\sigma_{em}}{\sigma_{ea}} + \dfrac{S_F}{S'_e}} \qquad (16)$$

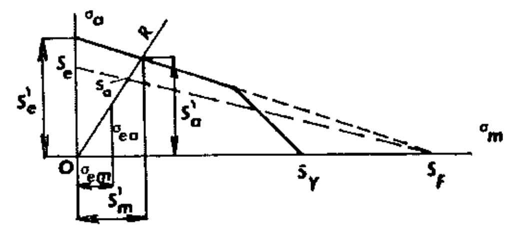

Fig. 4. Determination of Limiting Stresses in Combined Loading

and the limiting mean stress S'_m (S'_{tm} or S'_{bm})

$$S'_m = \frac{S'_e}{\dfrac{\sigma_{ea}}{\sigma_{em}} + \dfrac{S'_e}{S_F}} \qquad (17)$$

where S'_e is the endurance limit of the specimen under bending or axial load, respectively. In the actual part the modifying factors are different for each σ_1, σ_2, and σ_3 (Heywood 1962, Serensen 1975, Shigley 1983):

So the equivalent amplitude is

$$\sigma_{ea} = \left[(K_{1r}\sigma_{1a})^2 + (K_{2r}\sigma_{2a})^2 + (K_{3r}\sigma_{3a})^2 - (K_{1r}\sigma_{1a})(K_{2r}\sigma_{2a}) - (K_{2r}\sigma_{2a})(K_{3r}\sigma_{3a}) - (K_{3r}\sigma_{3a})(K_{1r}\sigma_{1a})\right]^{1/2} \quad (18)$$

Then

$$n = \frac{S_a}{\sigma_{ea}} \quad (19)$$

or

$$n = \frac{k_b\, S_a}{\sigma_{ea}} \quad (20)$$

The equation (19) is used when all stresses have a stress gradient equal to zero ($k_b = 1$). The equation (20) is used if at least one stress gradient in the analyzed point of the part is not equal to zero. The amplitude limit of the actual part used in Eq. 16 is

$$S_a = \frac{S_F}{\frac{\sigma_{em}}{\sigma_{ea}} + \frac{S_F}{S_e}} \quad (21)$$

It is also necessary to calculate the safety factor against yielding using the relationship

$$n_y = \frac{S_y}{\sigma_{emax}} \quad (22)$$

The equivalent maximum stress is

$$\sigma_{emax} = \sqrt{\sigma_{1max}^2 + \sigma_{2max}^2 + \sigma_{3max}^2 - \sigma_{1max}\,\sigma_{2max} - \sigma_{2max}\,\sigma_{3max} - \sigma_{3max}\,\sigma_{1max}} \quad (23)$$

where $\sigma_{1max} = \sigma_{1m} + \sigma_{1a}$, etc.

The mentioned formulae are also used in cases where the stresses are not in phase. The part will be overdesigned.

Combined Fluctuating Load with Unequal Coefficients of Nonsymmetry

In such case, the equations, 17, 18, 19, 20, 21, 22, 23, which are valid for equal R cannot be used directly. For this reason, it is necessary to transform all stresses with different coefficients of nonsymmetry R into stresses with equal coefficients.

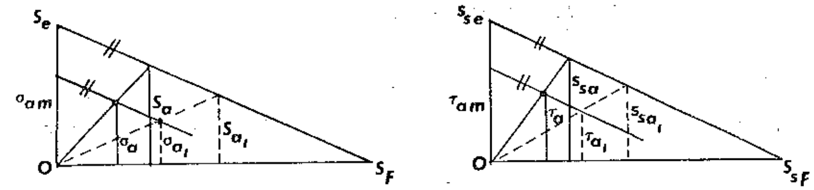

Fig. 5. Transformation of Cycles with Unequal R into Cycles with Equal R

Using Serensen's (1975) approach, the different stress cycles have the same effect, if they have the same safety factor. Graphically this transformation can be seen in Fig. 5, from which

$$n = \frac{S_a}{\sigma_a} = \frac{S_{a1}}{\sigma_{a_1}} \quad \text{or} \quad n = \frac{S_{sa}}{\tau_a} = \frac{S_{sa1}}{\tau_{a_1}} \quad (24)$$

More frequently, the cycles are reduced into symmetrically reversed cycles (in cycles with the coefficients of nonsymmetry R = -1). The transformed stresses can then be designated as σ_{am} (or τ_{am}). For the combined load in bending with amplitude σ_a and mean σ_m amd torsion (τ_a, τ_m) the design criterion using the distortion energy theory of a specimen is

$$n = \frac{S'_e}{\left[\sigma_{am}^2 + 3\tau_{am}^2\right]^{0.5}} \quad (25)$$

and of an actual part

$$n = \frac{S_e}{\left[(K_r\sigma_{am})^2 + 3(K_{sr}\tau_{am})^2\right]^{0.5}} \quad (26)$$

where S'_e or S_e are determined using Eqs. (1,5) and from the similarity of triangles in Fig. 5

$$\sigma_{am} = \tau_a + \tau_m \frac{S_e}{S_F}$$
$$\tau_{am} = \tau_a + \tau_m \frac{S_{se}}{S_{sF}} \quad (27)$$

DESIGN CRITERION FOR A SHAFT UNDER REVERSED BENDING AND STATIC TORSION

The coefficient of nonsymmetry for bending is R = -1 and for torsion is R = 1. Fig. 6 shows the transfer of shear stress τ_m in the reversed (symmetric) cyclic stress of amplitude τ_{am}.

From the similarity of triangles is

$$\frac{\tau_{am}}{\tau_m} = \frac{S_{se}}{S_{sF}}$$

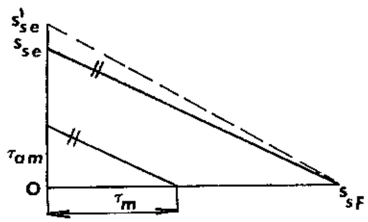

Fig. 6. Transformation of Shear Stress with R=1 into Shear Stress with R=-1

and then

$$\tau_{am} = \tau_m \frac{S_{se}}{S_{sF}} = \tau_m \frac{k_b k_c k_{sr} S'_{se}}{S_{sF}} \quad \text{or} \quad \tau_{am} = \tau_m \frac{k_b k_c S'_{se}}{K_{sr} S_{sF}} \tag{28}$$

The design criterion using the distortion energy theory is

$$\sigma_{ea} = \sqrt{(K_r\sigma_a)^2 + 3(K_{sr}\tau_{am})^2} = \sqrt{(K_r\sigma_a)^2 + 3\left(K_{sr}\frac{k_b k_c S'_{se}}{K_{sr}S_{sF}}\tau_m\right)^2} \le \frac{k_b k_c S'_{be}}{n} \tag{29}$$

In limited cases when the safety factor n = 1, then $\sigma_a = S_a$ and $\tau_m = S_{sm}$ the Equation (29) will be

$$\sqrt{(K_r S_a)^2 + 3\left(\frac{k_b k_c S'_{se}}{S_{sF}} S_{sm}\right)^2} = k_b k_c S'_{be}$$

and then

$$\frac{S_a^2}{\left(\frac{k_b k_c S'_{be}}{K_r}\right)^2} + \frac{(S'_{se}S_{sm})^2}{S_{sF}^2\left(\frac{S'_{be}}{\sqrt{3}}\right)^2} = 1$$

Because

$$\frac{k_b k_c S'_{be}}{K_r} = S_{be} \qquad \frac{S'_{be}}{\sqrt{3}} \approx 0.6\, S'_{be} = S'_{se}$$

then

$$\frac{S_a^2}{S_{be}^2} + \frac{S_{sm}^2}{S_{sF}^2} = 1 \tag{30}$$

Equation (30) is the equation of the ellipse with semiaxis S_{be} and S_{sF}, in Fig. 7 plotted as curve "1" (Puchner 1946). For practical use, it is necessary to restrict the maximum stress of ductible materials by yield strength.

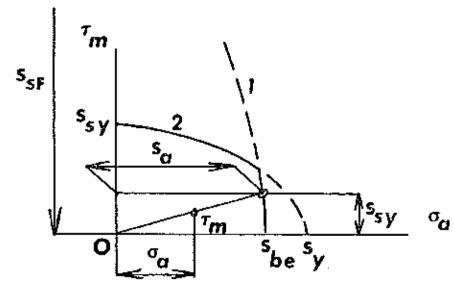

Fig. 7. Working Diagram for Combined Bending and Torsional Loading

Using the distortion energy theory the equivalent stress is

$$\sigma_{emax} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} \le \frac{S_y}{n}$$

where $\sigma_{max} = \sigma_a$ and $\tau_{max} = \tau_m$.

In cases when n = 1, it will be

$$\sqrt{S_a^2 + 3S_{sm}^2} = S_y$$

and

$$\frac{S_a^2}{S_y^2} + \frac{S_{sm}^2}{\frac{1}{3}S_y^2} = 1 \tag{31}$$

Equation (31) is the tquation of the ellipse with semiaxis S_y and

$$S_{sy} = \frac{S_y}{\sqrt{3}} \approx 0.6\ S_y$$

and in Fig. 7, it is designated as curve "2".

The point representing a given load must for $n > 1$ be in the zone limited by solid lines (Fig.7).

The design criterion for the given σ_a and τ_m is then determined by the relation

$$n = \frac{S_a}{\sigma_a}$$

CONCLUSION

This paper shows how to determine fatigue strength and the design criteria in a fluctuating load within the precision ± 10%. The described method is based on the evaluation of experimental data from different sources and gives to the designer a useful tool.

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