SYMMETRY AND COHERENCE IN QUANTUM MECHANICS

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ABSTRACT

In quantum mechanics, probability plays a fundamental role. As a consequence, the nature of the possible states for a translating particle can be induced from the prevailing symmetries over space, over time, and over state that it experiences. The forms of the standard commutation rules arise from an application of Lie-group theory to the *free* particle. One finds that over an infinitesimal increment of time, an observer cannot tell whether a particle in a steady state is interacting or not; so the same commutation rules apply and the symmetry considerations lead to a form for the Hamiltonian operator for a particle in *general*. However, a system containing independent parts is described by a density operator, not by a single ket.

A classical system can be subdivided into localized parts. Each of these parts sets up various fields. The fields from the different sources superpose to act on any given part. However, a configuration of the parts generally exhibits much less symmetry than the resulting potential energy and Lagrangian energy exhibit. Thus, a single planet moving in the field of an isolated star possesses a spherically symmetric potential. Nevertheless, the system is not spherically symmetric.

A coherent quantum system, on the other hand, cannot be similarly subdivided without altering its condition. As a consequence, the classical reduction in symmetry does not occur. The electron in the hydrogen atom moves in a spherically symmetric potential. And, the result may be a spherically symmetric atom, as in any *n*s state. A definite state that is not spherically symmetric, such as an *n*p state, is, on the other hand, a part of a degnerate irreducible representation of the full rotation group. A symmetry operation acting on an *n*p state yields a state that is a superposition of the three standard *n*p states.

Because the classical reduction in symmetry does not occur, the nature of definite states can be induced from symmetry considerations. A particle translating freely in a definite direction with a definite energy sees continuous symmetries over space, over time, and over its state function (Duffey, 1984a).

For a homogeneous beam travelling in the x-direction, we have

$$d\Psi = ik\Psi\,dx - i\omega\Psi\,dt. \qquad (1)$$

Integrating this yields

$$\Psi = A\,e^{ikx}e^{-i\omega t}. \qquad (2)$$

Thus, symmetry requirements lead one to the appropriate state function.

One can similarly treat a particle circling an axis with a definite angular momentum. The continuous symmetries determine the form of angular dependence of its state function (Duffey, 1984b). Indeed, these lead to

$$d\Phi = iM\Phi\,d\varphi. \qquad (3)$$

Integrating (3) gives us

$$\Phi = e^{iM\varphi} \qquad (4)$$

for the azimuthal state function. Symmetry considerations can be employed to obtain the possible corresponding colatitude state function $\Theta(\vartheta)$.

Alternatively, one can subject the free particle to a Galilean transformation. If we also assume that the particle is localizable, that it can act at a point in space and time, one is able to induce the commutation relation between the operators for position and momentum (Levy-Leblond, 1974; Duffey, 1986).

From the fact that the particle is localizable, we infer that an operator for position exists. If it is $\underline{X}$ before and $\underline{X}'$ after a translation by distance $\delta\underline{a}$, we have

$$X' = X + \delta a E. \qquad (5)$$

The infinitesimal operator for translation in space is $\underline{iP}$. So the translation by $\delta \underline{a}$ alters operator $\underline{X}$ by the similarity transformation

$$X' = (1 + iP\,\delta a)\,X\,(1 - iP\,\delta a)$$

$$= X - i[X, P]\,\delta a + PXP(\delta a)^2. \qquad (6)$$

On comparing (5) and (6), we obtain

$$[X, P] = iE \qquad (7)$$

in the natural units in which $\underline{\hbar} = 1$. Formula (7) is a form of the commutation relation between a linear coordinate and the corresponding momentum.

In most states of interest, any given particle is under influences from other particles. But here, a principle reminiscent of the principle of equivalence in general relativity comes into play. It is that, over an infinitesimal interval of time, an observer cannot tell whether a typical particle in a system is interacting with other particles or not. As a consequence, it is governed by the same commutation relations as the free particle.

These commutation relations impose invariances that give the Hamiltonian operator its conventional form (Levy-Leblond, 1974; Duffey, 1986). Consistent representations of the operators for positions, momenta, and energy can be constructed. Then one can formulate the conventional eigenvalue equations. These, of course, govern coherent systems.

A system containing two or more independent parts is not coherent, however. Such a system is described by a density operator, not by a single state function.

A typical macroscopic system is composed of a large number of independent microscopic parts. Within and among these, coherence may be broken down or formed without end. The governing laws are probabilistic. Statistical mechanics has been designed

to consider such systems and to derive their thermodynamic properties.

Small integrated systems, such as nuclei, atoms, molecules, domains of lattices, are coherent. On these, quantum mechanical calculations can be implemented. For systems made up of only a few particles, these are remarkably accurate.

Coherence also exists among the products of single disintegrations or decays. This persists until one of the products interacts with an initially independent particle. If such interaction does not occur before measurement, the coherence still exists then, even though the detectors may be meters apart (Aspect, Dalibard, Roger, 1982; Robinson, 1983).

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