TIME DILATION: A QUICK ANSWER

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When beginning physics students ask me to explain time dilation or the twin paradox, they don't have time or patience for the usual coordinate transformations and light cone diagrams.[1,2] I would like to share a rather striking and easily remembered statement which I use. A simple diagram can then be drawn to explain and calculate these effects.

The statement is: *Everything in the Universe is on a journey through space-time at the speed of light relative to any inertial reference frame.* "Relative to any inertial reference frame" is a standard phrase used to indicate that we are talking about a place where Newton's laws of motion are valid. The surface of the earth approximates an inertial reference frame. Any particle in the universe when plotted on a velocity diagram, relative to an observer on earth therefore, lies on the arc of a circle of radius equal to the velocity of light. The diagram, Fig. 1, has the magnitude of the spatial velocity, dr/dt, as abscissa and the proper time velocity, $cd\tau/dt$, as the ordinate.

Proper time, τ, is the time kept by the particle. For instance, it is the time read from the clock on the dashboard of a spaceship. The velocity, dr/dt, and time, t, are measured in an inertial reference frame.

With inspection of Fig. 1 it is easy to see that a particle at rest in space moves through proper time at the speed of light since it has no spatial component of velocity. The faster a particle moves through space the slower it moves through proper time. At the limit of a particle moving through space at the speed of light it does not move through proper time at all.

To point out the fact that there is a difference between the twins in the twin paradox the following story is helpful. One of a pair of twins travels to a star 10 light years distant at nearly the speed of light relative to his brother who stays home on the earth (the inertial reference frame of this story). He arrives at the star before his clock has ticked off more than a few seconds. He turns around quickly and returns at the same speed after a few more ticks of his clock. He hasn't even had time to eat the lunch he had packed for the journey. Meanwhile, his twin on the earth has been traveling through space-time at the same speed, however, all of his velocity is along the proper time coordinate. Both twins have traveled 20 light years in space-time relative to

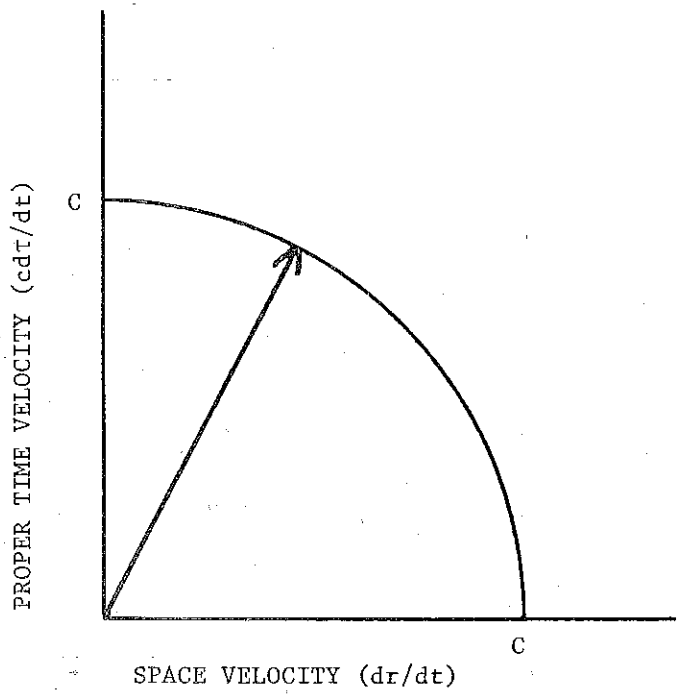

Figure 1. All particles in the universe including photons have a space-time velocity on the arc of the circle of radius c.

the inertial frame of the earth. One twin did most of his traveling in space, the other did most of his traveling in proper time. The one who traveled in proper time is obviously older, having eaten 7305 (20 x 365.25) lunches! Most students are satisfied by this explanation, and are pleased that it is simple enough to remember and relate to others.

If the student wants to know how this relates to more standard statements from the relativity theory, the definition of the invariant proper time interval can be used.[1,2]

$$c^2\, d\tau^2 = c^2\, dt^2 - dr^2$$

From this relationship it follows that

$$c^2(d\tau/dt)^2 + (dr/dt)^2 = c^2,$$

which is the mathematical equivalent of the above statement and diagram. Only the first quadrant of the circle in Fig. 1 is used since the magnitude of velocity is always positive and no speed will cause a clock to run toward negative time.

This explanation has the disadvantage that the velocity used here is not a four-vector under Lorentz transformation and is not what is referred to in the literature as the "four-velocity."[1,2,3] If the student later takes up the serious study of relativity, this explanation may lead to some confusion. On the other hand, the four velocity does not lend itself to simple diagrams and easy explanations. It has the nonintuitive feature that its square is a negative constant. Furthermore, the spatial components of the four-velocity are not limited by the speed of light which makes it confusing to the beginning student who has been told that nothing travels faster than the speed of light.

The hybrid four-velocity used here may not be useful in situations where the transformation properties of a four-vector are necessary, but it does serve a useful purpose in explaining time dilation simply and it stimulates interest. Students who learn that they are traveling through time at the speed of light typically respond with awe, "that's weird!"

REFERENCES

1. E. F. Taylor and J. A. Wheeler, **Spacetime Physics** (W. H. Freeman and Co., San Francisco, 1966) pp. 94-95, 112.

2. R. D. Sard, **Relativistic Mechancis** (W. A. Benjamin, Inc., New York, 1970) pp. 101-103, 118-121.

3. C. W. Misner, K. S. Thorne and J. A. Wheeler, **Gravitation** (W. H. Freeman and Co., San Francisco, 1973) pp. 49-50.