SIMULATION OF THE SOLAR INSTALLATION AT THE MT. RUSHMORE VISITOR CENTER *

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ABSTRACT

A mathematical model has been developed to simulate the detailed performance of the solar heating and cooling system which is being installed in the Mt. Rushmore Visitor Center. The model simulates the response of the building, the solar and conventional heating and cooling units, and the control system to weather data. Procedures used to minimize computation time and to ensure convergence are discussed. Predictions for the performance of the installation are also presented.

I. INTRODUCTION

The recent development of general simulation programs is a very powerful tool to assist in the design of complex heating and cooling systems which include solar components. Simulation in the context of this paper means that given the details of the system and the (hourly) weather data, the program is capable of predicting the detailed performance as a function of time. The simulation program will not design the system, find optimum sizes nor perform an economic analysis. The actual design must be based on the results of several simulations within the constraints of the particular application. For the Mt. Rushmore Visitor Center project, for instance, the constraints were the local geography, the shape and orientation of the building, the current heating and cooling systems and the use of the building. Discussion of these factors and how they relate to the actual design is explained in Section II.

The simulation program used here was "A Transient Simulation Program" (TRNSYS) prepared by the Solar Energy Laboratory, University of Wisconsin-Madison.[1] The intention of such a program is to enable the designer to describe the components of the system under consideration as a set of modules which are interconnected as in the actual system. Given the schematic of the system it should be possible for someone unfamiliar with computers to write a workable program. Section II, III, and IV contain a report of the author's experience with TRNSYS and the modifications in the basic program which were necessary to obtain higher accuracy in less time and to obtain convergent solutions. The predictions of the modified TRNSYS program for the Visitor Center installation are then presented in Section V.

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II. THE SYSTEM DESIGN

The design of the system was a cooperative effort involving primarily Drs. C. W. Chiang and R. L. Pendleton of the SDSM&T Mechanical Engineering Department, Mr. D. L. Rosenstein and Mr. K. Schmidt of the Spitznagel Partners, Inc., Mr. G. L. Merrill of Honeywell Energy Systems Center, and the author.

The first design decision was to install collectors which used liquid rather than air as the heat transfer fluid. There were three factors which indicated that this was the best choice: (1) A liquid medium would integrate easily with the existing system by simply installing a heating coil in the furnace air duct. The liquid could be circulated directly from the collectors to the heating coil for optimum transfer without additional blowers. (2) There is a need for extra air-conditioning in the observation room which could be met by solar activated lithium bromide adsorption refrigeration units. Air collectors do not produce high enough temperatures to activate these units. (3) The limited space to mount the collector array required collectors with high efficiency and air collectors are generally less efficient than liquid collectors.

A schematic of the solar system is shown in Fig. 1. The numbers on some of the components correspond to the unit numbers in the simulation program listed in Table 1. The sizes of the components and the control scheme are included as comments in the program listing, and are self explanatory.

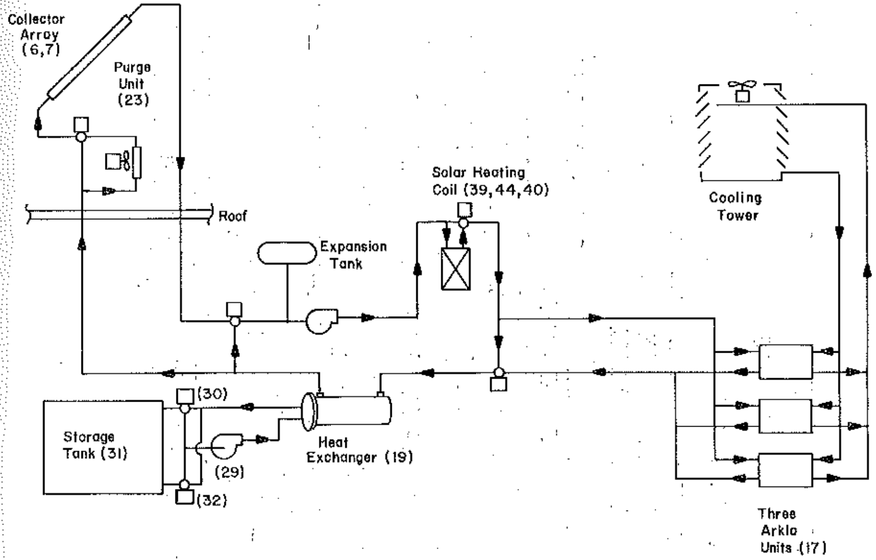

Figure 1. Solar System Schematic

TABLE 1

```
SIMULATION  0.000+000    8.760+003    5.000—001

LIMITS   30   15

WIDTH  120

TOLERANCES  3.000—003    3.000—003
*    VISITOR CENTER
*    YEAR OF 1971

UNIT 1     TYPE 9     CARD READER
     PARAMETERS 14
     7.000+000    1.000+000    4.000+000    4.186+001    0.000+000    5.000+000    5.150—001    0.000+000
     6.000+000    5.556—001   —1.778+001    7.000+000    5.556—001   —1.778+001
*    PAR4 = LANGLEY TO KILOJOULE/SQM CONVERSION
*    PAR7 = KNOTS TO M/SEC CONVERSION
*    PAR10, 11, 13, 14 = DEGF TO DEGC CONVERSION

UNIT 2     TYPE 16    SOLAR RADIATION PROCESSOR COLLECTORS
     PARAMETERS 8
     3.000+000    4.383+001    4.570+001   —1.880+001    4.871+003    1.000—001    0.000+000    8.760+003
     INPUTS 2
      0, 0         1, 4
     1.000+000    0.000+000
*    PAR2 = LATITUDE, PAR3 = SLOPE
*    PAR4 = ORIENTATION ANGLE, PAR5 = SOLAR CONSTANT
*    PAR6 = GROUND REFLECTANCE
```

Table 1 (Continued)

```
UNIT 3     TYPE 16    SOLAR RADIATION PROCESSOR WINDOW
     PARAMETERS 8
     1.000+000    4.383+001    9.000+001   —1.180+002    4.871—003    0.000+000    3.240+003    6.192+003
     INPUTS 2
      0, 0         1, 4
     1.000+000    0.000+000

UNIT 4     TYPE 16    SOLAR RADIATION PROCESSOR ROOF
     PARAMETERS 8
     1.000+000    4.383+001    9.300+000    6.200+001    4.871+003    0.000+000    0.000+000    8.760+003
     INPUTS 2
      0, 0         1, 4
     1.000+000    0.000+000

UNIT 5     TYPE 15    RADIATION AFTER SHADING
     PARAMETERS 10
     0.000+000    0.000+000    0.000+000    1.000+000    0.000+000    0.000+000    1.000+000    4.000+000
     8.000+000    4.000+000
     INPUTS 5
      2, 1         2, 1         0, 0         4, 1         0, 0
     0.000+000    0.000+000    7.679—001    0.000+000    1.664+000
*    HTS = HTC — (.7679*HTC — 1.664HTR)8
*    8 IS AN OPERATOR WHICH REPLACES A NEGATIVE NUMBER WITH ZERO

UNIT 6     TYPE 15    ALGEBRAIC  COLLECTOR QU
     PARAMETERS 12
     0.000+000    0.000+000    1.000+000    0.000+000    0.000+000    0.000+000    4.000+000    1.000+000
     4.000+000    8.000+000    0.000+000    1.000+000
```

Table 1 (Continued)

```
   INPUTS 6
    0, 0        5, 1        0, 0       23, 1        1, 6        0, 0
   7.800-001   0.000+000   1.396+001   2.000+001  -4.000+000   1.600+002
*  QU = (A*HT - B*(TIN - TA) )8*AREA

UNIT 7    TYPE 15    ALGEBRAIC  COLLECTOR TCO
   PARAMETERS 5
   0.000+000   0.000+000   0.000+000   2.000+000   3.000+000
   INPUTS 3
   23, 1        6, 1        0, 0
   2.000+001   0.000+000   2.112+004
*  TCO = TIN + QU/FL*CW

UNIT 11    TYPE 2    ON/OFF  DIFFERENTIAL CONTROLLER COLLECTOR
   PARAMETERS 3
   2.000+001   1.000+000   1.000+000
   INPUTS 3
    6, 1        0, 0       11, 1
   0.000+000   0.000+000   0.000+000
*  TURNS ON STORAGE TANK PUMP WHEN QU GT 1.0

UNIT 25    TYPE 14    TIME DEPENDENT DAILY THERMOSTAT
   PARAMETERS 12
   0.000+000   0.000+000   6.000+000   0.000+000   8.000+000   1.000+000   2.000+001   1.000+000
   2.000+001   0.000+000   2.400+001   0.000+000
*  TURNS THERMOSTAT DOWN AT NIGHT
```

Table 1 (Continued)

```
UNIT 36    TYPE 14    TIME DEPENDENT SEASONAL FORCING FUNCTION
   PARAMETERS 12
   0.000+000   1.000+000   3.240+003   1.000+000   3.240+003   0.000+000   6.192+003   0.000+000
   6.192+003   1.000+000   8.760+003   1.000+000
*  TURNED OFF FROM MAY 15 - SEPT 15

UNIT 26    TYPE 15    ALGEBRAIC  THERMOSTAT CONTROL
   PARAMETERS 10
   0.000+000  -1.000+000   1.000+000   0.000+000   4.000+000   0.000+000   1.000+000   0.000+000
   1.000+000   3.000+000
   INPUTS 4
   44, 4       25, 1       36, 1        0, 0
   2.000+001   0.000+000   0.000+000   8.000+000
*  TURNED DOWN BY 8 DEG C BEFORE MAY 15 AND AFTER SEPT 15

UNIT 27    TYPE 8    THREESTAGE ROOM THERMOSTAT
   PARAMETERS 6
   2.000+000   1.000+000   3.200+001   2.400+001   2.050+001   1.950+001
   INPUTS 2
   26, 1       31, 3
   2.000+001   2.000+001
*  PAR3 = MIN STORAGE TEMP FOR SOLAR HEAT
*  PAR4 = SETTING FOR AIR CONDITIONING
*  PAR5, 6 = SETTINGS FOR SOLAR AND FURNACE HEATING

UNIT 39    TYPE 11    FLOW DIVERTER SOLAR HEAT COIL BYPASS
   PARAMETERS 1
   2.000+000
   INPUTS 3
    7, 1        0, 0       27, 1
   2.000+001   5.933+003   0.000+000
```

Table 1 (Continued)

```
UNIT 42     TYPE 15     ALGEBRAIC  FURNACE
   PARAMETERS 1
   1.000+000
   INPUTS 2
   27, 2         0, 0
   0.000+000    2.500+005
*  QF = 250000*CONTROL

UNIT 37     TYPE 15     ALGEBRAIC  QAIR
   PARAMETERS 8
   0.000+000    0.000+000    1.000+000  -1.000+000    1.000+000    0.000+000    4.000+000    1.000+000
   INPUTS 3
   27, 3         0, 0         36, 1
   0.000+000    1.139+005    1.000+000
*  QAIR = 113940*CONTROL1*CONTROL2

UNIT 12     TYPE 14     TIME DEPENDENT Q LIGHTS AND PEOPLE
   PARAMETERS 12
   0.000+000    0.000+000    8.000+000    0.000+000    8.000+000    1.000+000    2.000+001    1.000+000
   2.000+001    0.000+000    2.400+001    0.000+000

UNIT 13     TYPE 15     ALGEBRAIC  QWINDOW - QAIR
   PARAMETERS 12
   0.000+000    0.000+000    1.000+000    0.000+000    0.000+000    1.000+000    4.000+000    8.000+000
   0.000+000    1.000+000    0.000+000    4.000+000
   INPUTS 6
    3, 1         0, 0         4, 1         0, 0         0, 0        37, 1
   0.000+000    1.004+002    0.000+000    2.010+001    8.000-001    0.000+000
*  QWINDOW - QAIR = (100.4*HTW - 20.1*HTR)8*0.8 = QAIR
```

Table 1 (Continued)

```
UNIT 43     TYPE 15     TOTAL HEAT TO LOAD
   PARAMETERS 20
   0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    4.000+000    0.000+000
   2.000+000    4.000+000    8.000+000    4.000+000    0.000+000    1.000+000    4.000+000    1.000+000
   0.000+000    3.000+000    0.000+000    3.000+000

   INPUTS 10
   12, 1         0, 0         0, 0         0, 0        44, 4         1, 6         0, 0         0, 0
   42, 1        13, 1
   0.000+000    3.300+004    1.670+000    1.670+000    2.000+001   -4.000+000    1.000+001    1.111+004
   0.000+000    0.000+000
*  Q = QMU + QLP + OF + QW - QAIR
*  QMU = (1.67 - (1.67 - (TR - TA)/10)8)*FF*FLCA, QLP = 33000*FF

UNIT 44     TYPE 12     SPACE HEATING LOAD
   PARAMETERS 8
   4.000+000    5.500+003    4.400+004    5.933+003    3.560+000    5.380-001    1.111+004    0.000+000
   INPUTS 4
   39, 3        39, 4         1, 6        43, 1
   2.000+001    5.933+003    2.000+001    0.000+000
   DERIVATIVES 1
   1.300+001
*  PAR2 = HEAT LOSS COEFFICIENT, PAR3 = HEAT CAPACITY OF LOAD
*  PAR4 = DOW THERM FLOW, PAR5 = DOW THERM SPECIFIC HEAT
*  PAR6 = HEAT COIL EFFECTIVENESS, PAR7 = AIR FLOW*SPECIFIC HEAT
```

Table 1 (Continued)

```
UNIT 40     TYPE 11     TEE PIECE SOLAR HEAT COIL BYPASS
   PARAMETERS 1
   1.000+000
   INUPTS 4
   44, 1        44, 2        39, 1        39, 2
   2.000+001    0.000+000    2.000+001    5.933+003

UNIT 10     TYPE 15     ALGEBRAIC SOLAR HEAT
   PARAMETERS 7
   0.000+000    0.000+000    4.000+000    0.000+000    1.000+000    0.000+000    1.000+000
   INPUTS 4
   39, 3        44, 1        0, 0         44, 2
   2.000+001    2.000+001    3.560+000    0.000+000
*  QS = (TIN - TOUT)*C*FLOW

UNIT 17     TYPE 7      ARKLA UNITS
   PARAMETERS 6
   0.000+000    0.000+000    0.000+000    1.139+005    3.240+003    6.192+003
   INPUTS 5
   40, 1        0, 0         -1, 6        1, 7         37, 1
   2.000+001    5.046+003    -4.000+000   -4.000+000   -0.000+000
*  PAR4 = NOMINAL CAPACITY, PAR5, 6 = TIME ON, OFF

UNIT 29     TYPE 15     ALGEBRAIC  STORAGE TANK PUMP
   PARAMETERS 17
   0.000+000    -1.000+000   1.000+000    0.000+000    4.000+000    1.000+000    0.000+000    3.000+000
  -1.000+000    1.000+000    0.000+000    4.000+000    1.000+000    0.000+000    3.000+000    0.000+000
   1.000+000
```

Table 1 (Continued)

```
   INPUTS 6
   11, 1        27, 1        27, 1        27, 3        27, 3        0, 0
   0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    6.808+003
*  FLOW = ((GO(1 - G1) + G1)*(1 - G3) + G3)*PR

UNIT 19     TYPE 5      HEAT EXCHANGER STORAGE TANK
   PARAMETERS 4
   4.000+000    6.392-001    3.560+000    4.190+000
   INPUTS 4
   17, 1        40, 2        32, 1        29, 1
   2.000+001    5.933+003    2.000+001    0.000+000
*  PAR2 = EFFECTIVENESS

UNIT 23     TYPE 13     PRESSURE RELIEF VALVE AND CONVERGER
   PARAMETERS 2
   1.000+002    3.560+000
   INPUTS 3
   19, 1        19, 2        19, 1
   2.000+001    0.000+000    2.000+001
*  MAXIMUM DOW THERM TEMP TO COLLECTOR = 100 DEG CELSIUS

UNIT 33     TYPE 2      ON/OFF DIFFERENTIAL CONTROLLER STORAGE TANK
   PARAMETERS 3
   1.000+001    0.000+000    0.000+000
   INPUTS 3
   19, 3        31, 3        33, 1
   2.000+001    2.000+001    0.000+000
```

Table 1 (Continued)

```
UNIT 21     TYPE 15     PRESSURE RELIEF VALVE
   PARAMETERS 6
   0.000+000    0.000+000    0.000+000    4.000+000    8.000+000    4.000+000
   INPUTS 3
   19, 3        19, 3         0, 0
   2.000+001    2.000+001    9.800+001
*  MAXIMUM WATER TEMP TO STORAGE =  98 DEG CELSIUS

UNIT 30     TYPE 11     FLOW DIVERTER UPPER/LOWER STORAGE TANK
   PARAMETERS 1
   2.000+000
   INPUTS 3
   21, 1        19, 4        33, 1
   2.000+001    0.000+000    0.000+000
*  HOT WATER IN TOP  COLD IN BOTTOM

UNIT 31     TYPE 4     STRATIFIED FLUID STORAGE TANK
   PARAMETERS 5
   1.125+001    1.500+000    4.190+000    1.000+003    8.600-001
   INPUTS 5
   30, 3        30, 4        30, 1        30, 2         0, 0
   2.000+001    0.000+000    2.000+001    0.000+000    1.500+001
   DERIVATIVES 3
   3.000+001    3.000+001    3.000+001
*  PAR1 = TANK VOLUME (CU METERS), PAR5 = INSULATION HEAT LOSS
```

Table 1 (Continued)

```
UNIT 32     TYPE 11     TEE PIECE UPPER/LOWER STORAGE TANK
   PARAMETERS 1
   1.000+000
   INPUTS 4
   31, 1        31, 2        31, 3        31, 4
   2.000+001    0.000+000    2.000+001    0.000+000

UNIT 45     TYPE 24     INTEGRATOR MONTHLY
   PARAMETERS 1
   7,300+002
   INPUTS 8
   10, 1        42, 1         2, 1         5, 1        23, 3        17, 3        17, 4        37, 1
   0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    0.000+000    0.000+000

UNIT 47     TYPE 25     PRINTER MONTHLY
   PARAMETERS 3
   7.300+002    7.300+002    8.760+003
   INPUTS 8
   45, 1        45, 2        45, 3        45, 4        45, 5        45, 6        45, 7        45, 8
   QS           QF           HT           HTS          QR           TCOOL        ACOOL        QCOOL
```

To fully describe the rationale for each design decision is not the purpose of this paper, however, some of the more important decisions were determined according to the following conditions: (1) The collector area (160 m²) is the maximum which can be fit on the roof and allow enough spacing to avoid shading (5% of total in December). (2) The three Arkla Units (Unit 17) are the maximum number which can be operated with the given collectors. (3) The 11.25 m³ (3000 gal.) storage tank (Unit 31) is a compromise between performance and available space. (4) The solar heat coil, (Parameter 6 of Unit 44) effectiveness = 0.538, is the largest reasonable coil which can fit into the existing furnace duct. (5) The angle at which the collectors are placed (Unit 2) is a combination of the orientation of the roof, a 45° angle relative to the roof for ease of construction, and a need for solar collection both winter and summer. The optimum angle for this particular application was not determined, but other, simpler models[3] show that the performance is probably only 2% less at this angle than at the optimum of about 55°. The angles used here favor the air conditioning application. (6) The control system is designed to use solar heating or cooling first and then the current conventional systems as backup. It will be possible to air condition and solar heat during the same day even though this feature is not in the computer model for simplification.

III. THE SIMULATION MODEL

The TRNSYS program used here should enable the user to write a workable transient simulation program with a minimal knoweldge of computers. TRNSYS has built-in checks to spot obvious errors such as calling for an input from a nonexistent unit, failure to specify initial conditions, etc. Other errors such as improper conversion of units, crossing the temperature with the flow rate, etc., can be very difficult to spot. In this case the user must know enough about the system to realize that the answers are nonsense, even though the computer processes the numbers without printing an error message. If the program fails to produce answers which converge within the specified tolerances in the specified number of iterations, the user has recourse to decreasing the time step, raising the tolerances of number of iterations, or sticking the control units after fewer calls in one time step. When these fail to produce convergence, then it is essential to have familiarity with FORTRAN and general knowledge of iterative techniques to trace and correct the difficulty.

The program of Table 1 was generated by applying the TRNSYS manual to the schematic diagram of Fig. 1. However, the resulting program did not converge due to instabilities inherent in the program. Other instabilities and slow convergences had to be removed to achieve shorter calculation times with high accuracy.

IV. INSTABILITIES AND SLOW CONVERGENCES

One important instability in the original program arose from the fact that the Arkla unit did not have a way to stick it in either the on or the off position. The following situation would frequently occur in a series of iterations during a time step: (1) The collector output would be hot enough to operate the Arkla unit. (2) With the unit on, the temperature of the fluid returning to the collector would be lowered. (3) The resulting collector output would not be hot enough to operate the Arkla unit. (4) With the Arkla unit off, the temperature of the fluid returning to the collector would not be lowered. (5) The collector output would again be hot enough to operate the Arkla unit, and so on. The basic program was altered so that if the Arkla unit was off for any iteration during a time step, then it would stay off until the next time step. This unit was also altered so that calculations of the cooling load and Arkla performance characteristics would be bypassed during specified time periods when air conditioning is obviously not needed.

An example of a possible slowly converging situation is a solar collector connected to a heat exchanger as shown in Fig. 2. For simplicity it will be assumed that the fluid entering the cold side of the heat exchanger is at constant temperature, T_C. The following relationships hold:

$$T_H - T_I = \varepsilon (T_H - T_C) \qquad (1)$$

where ε is the effectiveness of the heat exchanger, T_H is the temperature of the

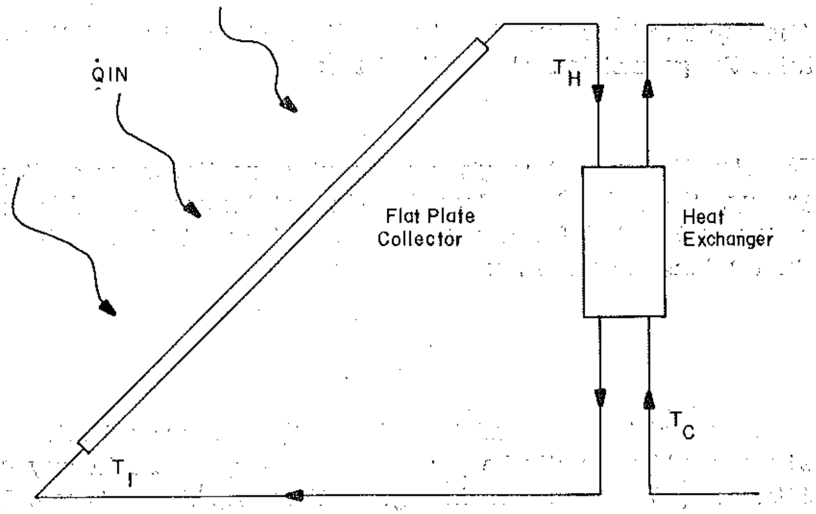

Figure 2. Solar Collector — heat exchanger schematic

hot fluid leaving the collector and T_I is the temperature of the fluid entering the collector.

$$T_H = T_I + \dot{Q}_u/\dot{m}c \qquad (2)$$

$$\dot{Q}_u = A\,\dot{Q}_{IN} - B\,(T_I - T_A) \qquad (3)$$

where $\dot{Q}_u$ is the useful rate of heat collection, $\dot{m}$ is the mass flow rate of collector fluid, $\dot{Q}_{IN}$ is the solar insolation, T_A is ambient temperature, and A and B are performance characteristics of the collector.

These equations can be rewritten to give the outputs of the two units. For the heat exchanger:

$$T_I = (1 - \varepsilon)\,T_H + \varepsilon T_C \qquad (4)$$

For the solar collector:

$$T_H = \left(1 - \frac{B}{\dot{m}c}\right) T_I + \frac{A}{\dot{m}c}\,\dot{Q}_{IN} + \frac{B}{\dot{m}c}\,T_A \qquad (5)$$

After several iterations it can be established that the solution for T_I is given by:

$$T_I = \varepsilon T_C + \frac{A' + B'\ \varepsilon T_C}{B'} \sum_{i=1}^{\infty} [B'(1-\varepsilon)]^i \qquad (6)$$

where $A' = A\dot{Q}_{IN}/\dot{m}c + (B/\dot{m}c)\,T_A$, and $B' = 1 - B/\dot{m}c$.

The important consideration here is the fact that the solution contains the geometric series in the form:

$$S = C + D \sum_{i=1}^{\infty} r^i \qquad (7)$$

The iterative procedure gives consecutive partial sums as it progresses. It can be shown by using the well known sum of the geometric series[3] that for any three consecutive partial sums, S_1, S_2, S_3 the total sum is given by:

$$S = \frac{S_1 S_3 - S_2^2}{S_1 + S_3 - 2S_2} \qquad (8)$$

This fact has been written into the "pressure relief valve" unit which would normally be present in such a system in any case. After every third iteration in a given time step this unit calculates the value of S from Eqn. (8) and substitutes it for S_3, the last previous value of the partial sum. Since this is the exact solution, this value will be repeated at the next iteration and the simulation will move on to the next time step. Thus, loops of this type will converge to arbitrarily high accuracy in four iterations.

It is instructive to look at some typical numbers for the given example. Suppose $\varepsilon = 0.7$ and $B/\dot{m}c = 0.08$, then $r = 0.276$. At the third call in the time step the r^2 term is added which has a relative precision of $0.276^2 = 7.6\%$. However, after using Eqn.(8) the exact solution is given at the third call. In the case of the primary loop of the Visitor Center simulation, there were two heat exchangers which made the convergence even slower. The principle developed here is valid for any number of heat exchangers.

It should be noted that the slowness of convergence of this example is not due to the size of the time step. No matter how small the step, if a unit switches on or off, it will require a relatively large difference of balance point for the temperatures. The size of this difference and the value of r determine the number of iterations required to achieve a given precision.

An example of a possible slowly converging or a diverging situation is a fluid entering a storage tank at a fixed temperature T_I, as shown in Fig. 3. The rate of change of temperature of the fluid in the tank, T, assuming thorough mixing, is given by:

$$\dot{T} = (T_I - T)\,\dot{m}/M \qquad (9)$$

where $\dot{m}$ is the rate of fluid flow in and M is the amount of fluid in the tank. Note that $M/\dot{m}$ is the time ,τ, it would take to fill the tank at the given flow rate.

The method of integration used by TRNSYS is the following:

$$T_1 = T_o + \dot{T}_o\,\Delta t \qquad (10)$$

where T_o and $\dot{T}_o$ are the temperature and its derivative at the end of the previous time step and Δt is the time step.

$$T_2 = T_o + \frac{\Delta t}{2}\,(\dot{T}_o + \dot{T}_1) \qquad (11)$$

where $\dot{T}_1$ is the value of $\dot{T}$ evaluated by Eqn.(9) with $T = T_1$. The net result for $N \geq 2$ is:

$$T_N = T_o + (T_I - T_o + \tau\dot{T}_o)\sum_{i=1}^{N-1}\left(\frac{\Delta t}{2\tau}\right)^i(-1)^i + 2\tau\dot{T}_o\left(\frac{\Delta t}{2\tau}\right)^N(-1)^{N-1} \qquad (12)$$

$$T_\infty = \frac{T_o + (T_I + \tau\dot{T}_o)\,\Delta t/2\tau}{1 + \Delta t/2\tau} \qquad \text{for } \frac{\Delta t}{2\tau} < 1. \qquad (13)$$

This iteration also produces a sequence of partial sums of a geometric series to which Eqn. (8) can be applied to obtain the exact solution. The last term of Eqn. (12) does not affect the result as long as the first sequence containing T_o is not used. An operation of the form of Eqn. (8) has been added to the general program to apply to all cases where differential equations are used in the simulation.

It should be noted that the convergence of this series depends on the time step. If the time step is twice as large as the characteristic time of the system, then the iteration will not be convergent. However, the application of Eqn. (8) still produces the result of Eqn. (13). A physical interpretation of Eqn. (13) can be understood

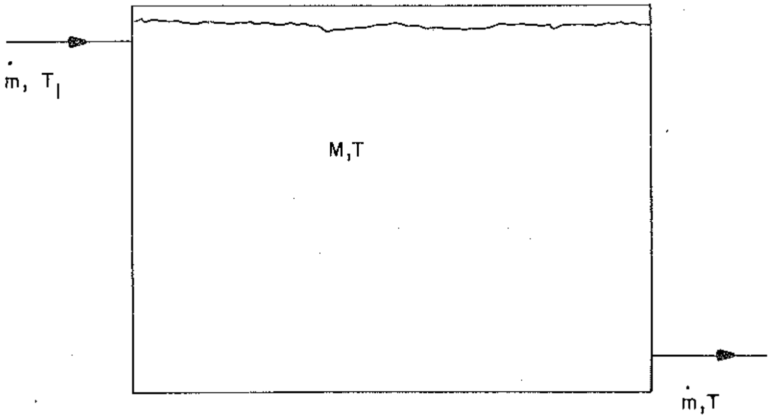

Figure 3. Storage tank with constant inlet temperature

TABLE 2

SOLAR SYSTEM PERFORMANCE — UNITS OF 10^6 KJ

Month	Heat Load	Solar Heat	% Solar	Cooling Load	Solar Cooling	% Solar
January	91.6	20.4	22	......	......	...
February	81.5	35.7	44	......	......	...
March	59.4	34.4	58	......	......	...
April	31.0	21.5	69	......	......	...
May	19.2	16.5	86	4.4	1.1	25
June	4.3	4.3	100	20.0	9.4	47
July	......	......	......	24.5	13.3	54
August	......	......	......	28.8	16.4	57
September	17.2	13.1	76	7.3	5.0	69
October	33.4	20.6	62	......	......	...
November	56.0	23.7	42	......	......	...
December	79.0	21.0	27	......	......	...
Total	472.6	211.2	45	85.0	45.2	53

as the resulting temperature when two like fluids of temperature T_o and $T_I + {}_\tau\dot{T}_o$ respectively are mixed in the ratio of 1 to $\Delta t/2\tau$. The quantity ${}_\tau\dot{T}_o$ is the difference between the inlet temperature at the previous time step and the storage temperature. Therefore, even in a situation where the iteration would diverge, the application of Eqn. (8) gives a reasonable result. Since this gives the exact answer, the next iteration will duplicate the answer (even for a divergent case) terminating the iterations in that time step.

These modifications of the TRNSYS program have proven to be very valuable in producing simulation models which converge rapidly. Since most heat flow problems involve linear relationships between the heat flow rate and temperature differences, Eqn. (8) is generally valid. It should be added that if the denominator of Eqn. (8) is less than 10^{-5}, the program sets S equal to the last partial sum, S_3, to avoid errors due to loss of precision in the differences of large numbers.

V. THE PREDICTED PERFORMANCE

The calculated performance of the Mt. Rushmore Visitor Center solar installation for the data of 1971 taken at the Rapid City Airport is shown in Table 2. The year 1971 was chosen as relatively typical (average temperature 1°C low, insolation about 2% low). It is expected that the air conditioning performance will be somewhat better than shown here due to the fact that Arkla has modified the unit to operate at lower temperatures and with higher COP. The performance specifications for the new Arkla units are being put into analytical form for substitution into the program.

The installation is scheduled to be in operation early in the summer of 1977. It will be instructive to compare the actual performance with the predictions of 45% of the heating load and 53% of the cooling load carried by the solar system. This will provide an excellent test of simulation techniques used in this work.

The transient performance of the system is not displayed here, but it has been calculated. The system will be fully instrumented to measure all temperatures and flow rates of importance as well as weather data at desired time intervals. This transient behavior will be compared with the simulation program.

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