INITIAL INELASTIC SURFACE DISTURBANCES RESULTING FROM UNDERGROUND EXPLOSIONS — QUASI-STATIC MOTION

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ABSTRACT

An analytical expression is obtained for the underground explosion pressure required to cause the first inelastic disturbances to appear at the ground surface. The motion is supposed to be quasi-static and spherically-symmetric. The medium is assumed to behave in a rigid-perfectly-plastic manner and to obey a linear yield criterion. Solutions are formulated for a material exhibiting both internal friction and cohesion, for a cohesionless material, and for a frictionless material. Quantitative agreement with available experimental data for shallow contained explosions in various rock types is quite encouraging in view of the numerous restrictive assumptions employed throughout the analysis.

INTRODUCTION

If an explosive charge is placed in an artificially created cavity at a given depth below the ground surface, in general one of three effects will be observed depending upon the size and depth of the charge and the type of rock. In particular, either the rock will be broken and ejected upward and in so doing form a crater; or the surface will suffer large upward inelastic displacements with or without visible fractures such that a crater is not formed; or finally the surface will undergo small upward elastic displacements without visible failure of the rock. It is the second of these effects which is of interest at this time. More exactly, we would like to be able to calculate the pressure required to just cause initial inelastic disturbances to appear at the ground surface for a given charge depth and cavity size.

The detonation of a contained explosive subjects the surrounding medium initially to a dynamic impulse and secondly to a quasi-static pressure resulting from the expansion of the residual gaseous products of the explosion. If the medium is highly dissipative such that the transient waves generated by the explosive impulse are quickly attenuated, then the behavior of the medium under the influence of the quasi-static gas pressure is of primary concern. In particular, soils, volcanic tuff, chalk rock, and porous sandstones or limestones are probably reasonably good examples of materials which dissipate transient disturbances fairly rapidly and, consequently, would be appreciably affected only by the quasi-static nature of the explosion.

STATEMENT OF THE PROBLEM

The problem can be defined as follows: Given a spherical cavity of radius "a" situated at a depth "h", find the pressure "P" which will first cause inelastic disturbances to appear at the ground surface. The pressure is assumed to be applied uniformly to the cavity wall in a quasi-static fashion such that inertial effects can be ignored. The medium is supposed to be isotropic and homogeneous, and to behave in a rigid-perfectly-plastic manner; i.e., the elastic strains are assumed negligible as compared to the plastic strains and the material exhibits no strain-hardening. Furthermore, it is supposed that the onset of plastic flow in the medium is governed by a linear yield criterion which may be specialized to include situations for which the yield point of a material may or may not be linearly dependent on hydrostatic pressure. In general the above assumptions concerning the behavior of the material are not physically unrealistic for soils or rock.

REVIEW OF THE ASSUMPTIONS

In practice it is virtually impossible and unfeasible to excavate a spherical cavity, particularly in situations requiring multiple blastholes. However, if the length of the explosive charge is of the order of the charge diameter, then an assumption of spherically-symmetric motion should be fairly realistic.

The pressure-time relationship for a contained explosion is characterized by an initial pressure spike which decays rapidly to a fairly stable constant "quasi-static" pressure as shown in Figure 1 (a). The quasi-static pressure P is approximately 0.4 of the peak initial dynamic pressure P_o and is maintained for a relatively longer time. Consequently, the assumed form of the quasi-static pressure pulse shown in Figure 1 (b) is within reason.

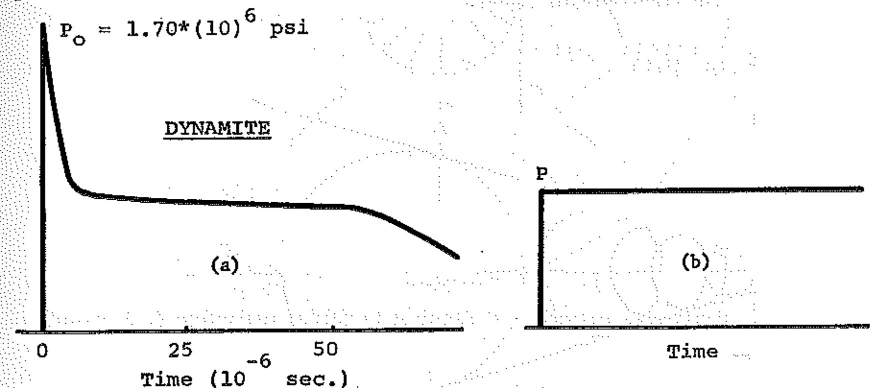

Figure 1. (a) Actual (1)* and (b) idealized explosion pressure pulses as functions of time.

* Number in parenthesis designates the reference.

If a material can experience large plastic strains as compared to the elastic strains and exhibits no strain-hardening, as is the case for the schist shown in Figure 2(a), then a rigid-perfectly-plastic assumption for the stress-strain relationship is not unrealistic. Furthermore, the failure envelopes of many rock types are approximately linear as, for example, is shown in Figure 3. In this instance, as in the case for many rock types, the yield point is linearly dependent on hydrostatic pressure.

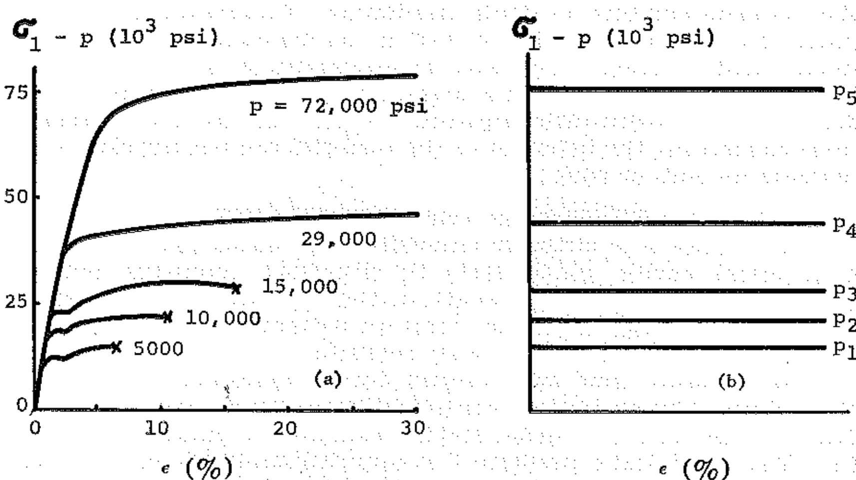

Figure 2. (a) Stress-strain curves for Virginia Greenstone (Schist) at various confining pressures (2) and (b) idealized rigid-plastic stress-strain curves.

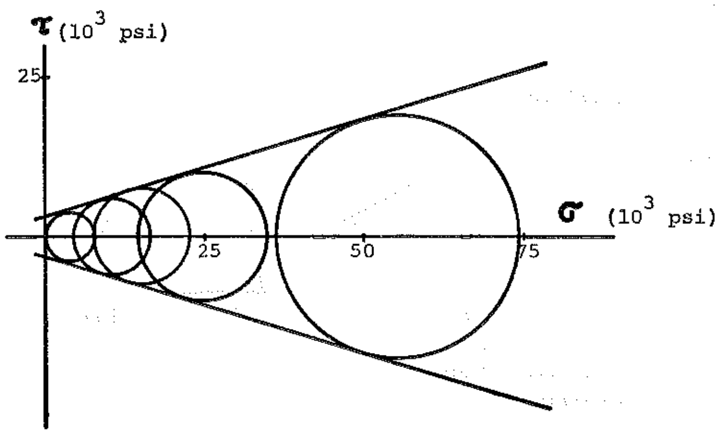

Figure 3. Failure envelope for Virginia Greenstone (Schist) (2).

ANALYSIS

The state of stress around and in the vicinity of the vertical axis passing through the center of the cavity may be approximately determined by tracing a sphere which is concentric with the center of the cavity and tangent to the ground surface and by replacing the vertical gravity field with vectors of uniform magnitude converging toward the center of the cavity (i.e., by assuming a hydrostatic field stress). Since the problem is supposed to be spherically-symmetric the unknown stress components are governed by one equation of equilibrium;

$$\frac{d\sigma_r}{dr} + \frac{2(\sigma_r - \sigma_\theta)}{r} + R = 0 \qquad (1)$$

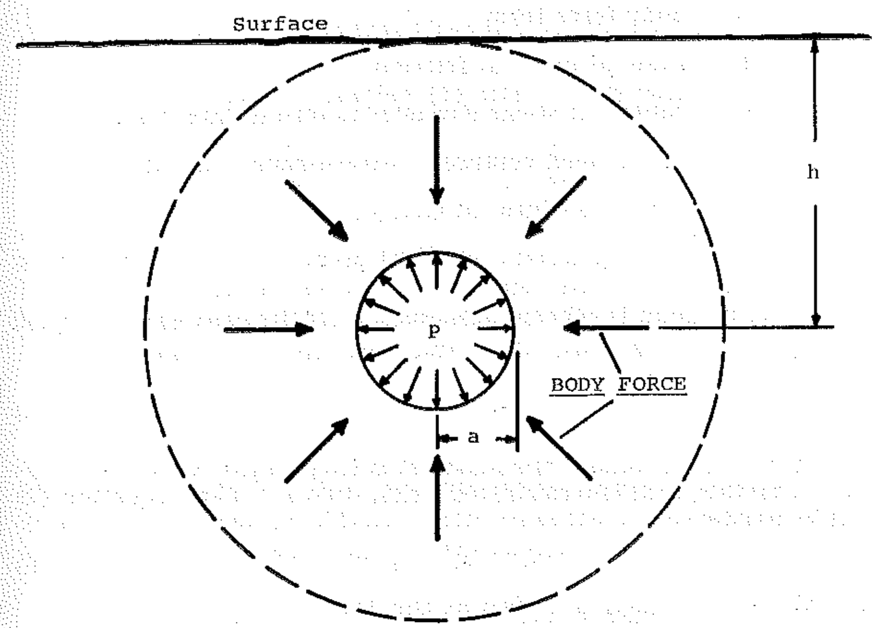

Figure 4. A spherical cavity at a depth h subjected to a static pressure P.

the Mohr-Coulomb yield criterion written in terms of principal stresses:

$$\sigma_r - k\sigma_\theta = Y \tag{2}$$

where $k = \dfrac{1 + \sin\phi}{1 - \sin\phi}$

$$Y = \frac{2\tau_o\cos\phi}{1 - \sin\phi} = \sigma_o$$

and the boundary condition:

$$\sigma_r = 0 \text{ when } r = h \tag{3}$$

where r,θ = radial and tangential coordinates

R = body force term

ϕ = angle of internal friction

τ_o = cohesion; shear strength at zero normal stress

σ_o = unconfined compressive (crushing) strength

h = depth to center of cavity

ϕ, τ_o, and σ_o are characteristic physical properties of soils and rock.

If the medium is cohesionless, such as is the case with dry sand or fragmented rock, then $\tau_o = 0$ and equation (2) becomes:

$$\sigma_r = k\sigma_\theta \tag{4}$$

On the other hand if the material is frictionless, as is characteristic of metals, salt, and saturated clays, then $\phi = 0$ and equation (2) can be written:

$$\sigma_r - \sigma_\theta = Y \tag{5}$$

The above expression is known as the Tresca yield criterion.

Notably, the yield criteria given by equations (2) and (4) require that the yield strength of the material be dependent on hydrostatic pressure; however, equation (5) assumes Y to be independent of hydrostatic pressure. The Mohr failure envelope for each case is shown in Figure 5.

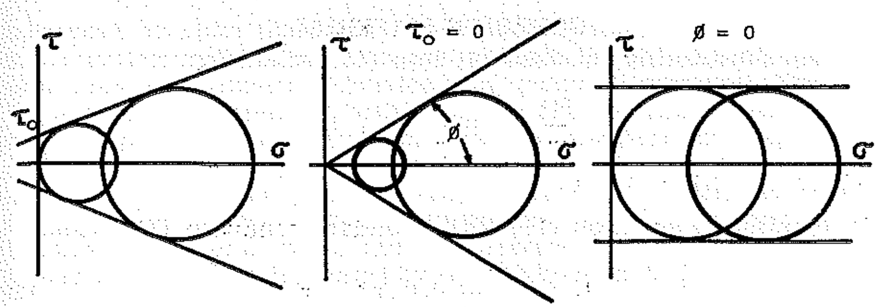

Figure 5. Various limiting conditions of the Mohr-Coulomb yield criterion.

In the following, solutions to the problem will be formulated for a material possessing both internal friction and cohesion, for a cohesionless material, and for a frictionless material.

MATERIAL WITH COHESION AND INTERNAL FRICTION

By substitution of equation (2) into (1), we obtain:

$$\frac{d\sigma_r}{dr} + n\frac{\sigma_r}{r} = -R - \frac{2Y}{kr} \tag{7}$$

where $n = 2(k - 1)/k$

Upon integration of the above equation and consideration of the boundary condition (3), it follows that:

$$\sigma_r = \frac{Y}{k-1}\left((h/r)^n - 1\right) + \frac{\rho}{n+1}\left(h(h/r)^n - r\right) \tag{8}$$

where R is interpreted as the density of the medium ρ. The pressure required to cause the first inelastic disturbances to appear at the ground surface can be obtained from equation (8) by letting $\sigma_r = P$ at $r = a$, where a is the radius of the cavity:

$$P = \frac{Y}{k-1}\left((h/a)^n - 1\right) + \frac{\rho}{n+1}\left(h(h/a)^n - a\right) \tag{9}$$

For near surface underground explosions, say for $h \leqq 100a$, the last term in equation (9) can be neglected and consequently:

$$P = \frac{Y}{k-1}\left((h/a)^n - 1\right) \tag{10}$$

COHESION MATERIAL

By substituting equation (4) into (1), we obtain:

$$\frac{d\sigma_r}{dr} + n\frac{\sigma_r}{r} = -R \qquad (11)$$

where $n = 2(k - 1)/k$

Integration of equation (11) with boundary condition (3) yields:

$$\sigma_r = \frac{\rho}{n + 1}\left(h\,(h/r)^n - r\right) \qquad (12)$$

Inelastic disturbances will occur at the ground surface when we let $\sigma_r = P$ at $r = a$ in equation (12); viz.:

$$P = \frac{\rho}{n + 1}\left(h\,(h/a)^n - a\right) \qquad (13)$$

The above expression was originally found by A. Nadai (3) under the assumption that the cavity was situated in a layer of gravitating, cohesionless material.

FRICTIONLESS MATERIAL

Equation (5) combined with equation (1) gives:

$$\frac{d\sigma_r}{dr} = -\frac{2Y}{r} - R \qquad (14)$$

By integration of the above expression it follows that

$$\sigma_r = 2Y\ \ln(h/r) + \rho(h-r) \qquad (15)$$

where use has been made of the boundary condition (3). When $\sigma_r = P$ at $r = a$, equation (15) can be written:

$$P = 2Y\ \ln(h/a) + \rho(h-a) \qquad (16)$$

Hence, the above expression permits the calculation of the pressure necessary to cause initial inelastic surface disturbances in a frictionless material.

DISCUSSION

Equations (9), (13), and (16) have been evaluated for a variety of cavity radii for depths of 10 and 100 feet with physical property data which are somewhat characteristic of soil and rock (see Figure 6). As is to be expected, the explosion pressure necessary to cause the first inelastic disturbances to appear at the surface of a cohesionless material is strongly dependent on depth; for an increase in depth by an order of magnitude, the pressure must be

increased by more than two orders of magnitude. Conversely, for a frictionless material an increase in depth by an order of magnitude only approximately doubles the required pressure. Finally, for a material possessing both cohesion and internal friction, the required pressure increase is comparable to the order of magnitude of the increase in depth. Depending, of course, upon the angle of internal friction, the pressure requirement for a material with cohesion and friction increases, in general, less rapidly than for a frictionless material when the compressive strengths are maintained identical and increased by equal amounts.

If explosive charges of constant weight and type are placed at different depths in the same material and detonated separately, a plot of crater volume (or amount of rock broken completely free from the rock mass) versus depth to the center of the charge yields a somewhat bell-shaped curve as shown in Figure 7. The minimum explosive charge depth corresponding to a zero crater volume and possibly to a situation involving inelastic surface disturbances is designated by h_o in the Figure. Some experimental data are available from cratering experiments in which curves of the above type were obtained (4,5). In particular, Table 1 is a comparison of explosion pressures calculated from equation (1) and (16) and those given in the above references (as calculated from the hydrodynamic theory of detonation) for five rock types.

TABLE 1

Rock	$Y = \sigma_o$ (10^3 psi)	h (ft.)	a (in.)	$P_{expl.}$ (10^6 psi)	P_{static} (10^6 psi)	$P_{calc.}$ (10^6 psi)
Chalk	2.0	6.2	1.50	0.99	0.40	0.18
"	2.0	4.7	1.02	0.99	0.40	0.21
Sandstone	10.0	6.9	2.76	0.99	0.40	0.46
Marlstone	10.0	6.0	1.50	0.99	0.40	0.87
"	10.0	6.0	1.50	0.99	0.40	0.77*
"	10.0	4.7	1.02	0.99	0.40	1.07
Granite	30.0	5.0	1.98	0.99	0.40	1.21
Salt	31.8	6.1	2.50	0.96	0.38	0.33**
"	31.8	6.1	2.50	0.96	0.38	0.21*

Note: $P_{expl.}$ are from references (4,5); $P_{static} = 0.4\ P_{expl.}$; all $P_{calc.}$ are calculated from equation (1) assuming $\phi = 30^o$ except ** for which $\phi = 5^o$ and except * which are calculated from equation (16) for which $\phi = 0^o$. In all instances the body force contribution is assumed to be negligible.

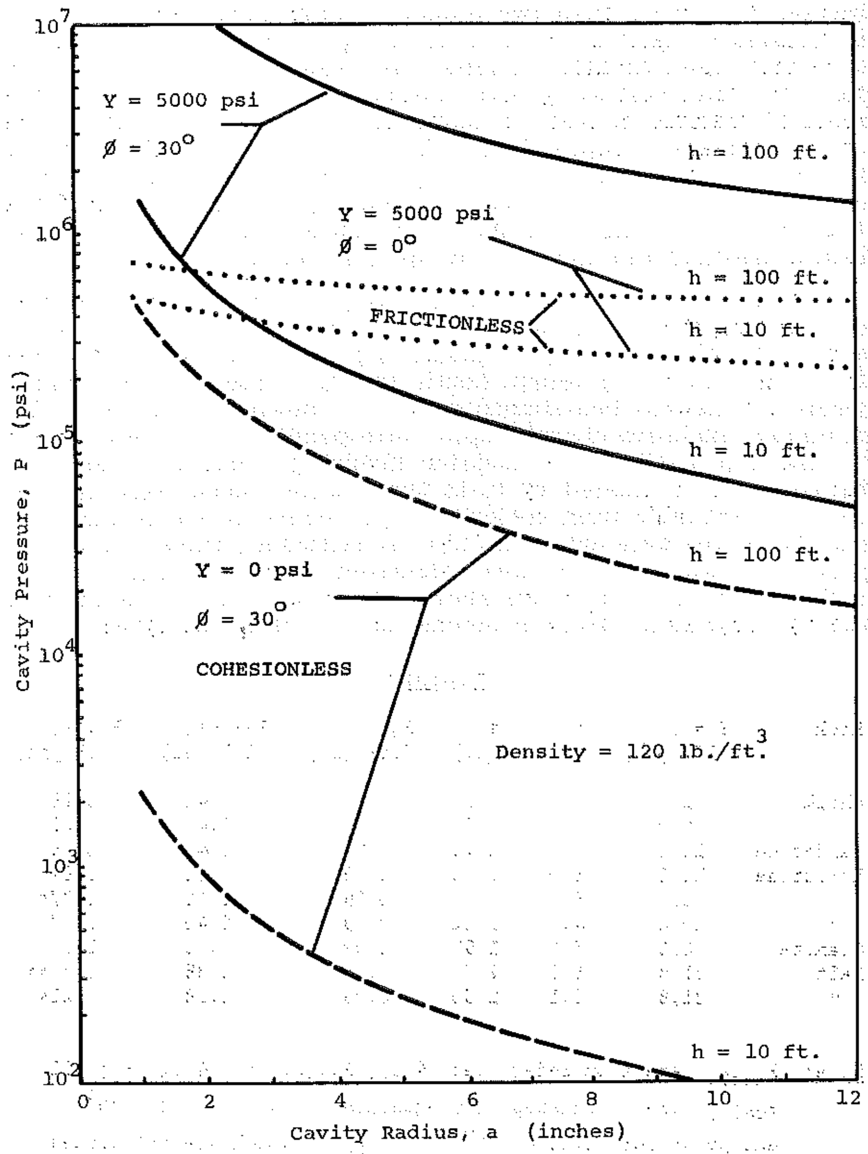

Figure 6. Explosion pressure as a function of cavity radius.

For rocks which will generally undergo considerable plastic deformation at low confining pressures in triaxial compression experiments, such as salt, sandstone, etc. (6,7), the quantitative agreement is remarkably encouraging. In spite of the fact that the explosion chambers were not spherical, but rather cylindrical, and that angles of internal friction were assumed, the derived formulae appear to yield explosion pressure values of the same orders of magnitude as calculated from those given in the references.

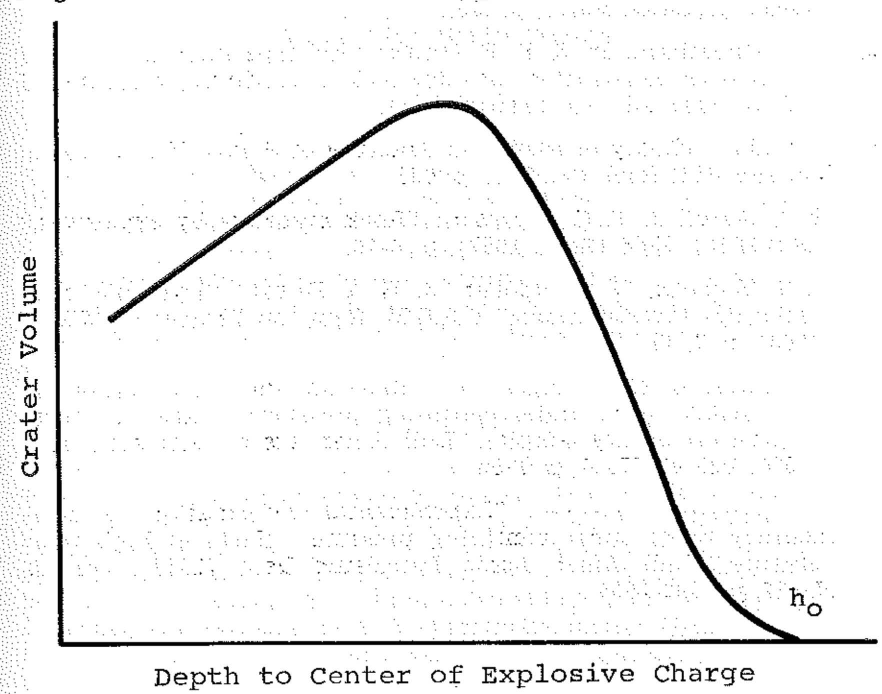

Figure 7. A characteristic plot of crater volume versus depth to center of explosive charge for single contained explosions in rock.

CONCLUSIONS

Analytical expressions are derived for the underground explosion pressures required to cause the first inelastic disturbances to appear at the ground surface for materials obeying specific forms of the Mohr-Coulomb yield or failure criterion. The motion is assumed to be quasi-static and spherically symmetric. The quantitative agreement with available experimental data for shallow contained explosions in various rock types is quite encouraging in view of the numerous limiting assumptions employed throughout the analysis. The formulae appear to be valid at least to a first approximation for materials which are generally known to exhibit considerable plastic deformation at low confining pressures in triaxial compression experiments.

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