LOW LYING ENERGY LEVELS OF Pb^{205}

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Pb^{203} is magic in both neutron and proton number and so is particularly stable. Pb^{205} has three fewer neutrons than Pb^{208}, and its low lying energy levels can be adequately described by an interacting system of three "neutron holes."

When examing nuclear phenomena, it is often helpful to make analogies with similar electronic phenomena. Bacher and Goudsmit[1] have presented a relatively simple method of predicting approximate energy levels for several electrons in a central field. This paper is an attempt to apply their procedure to a nuclear situation, in particular to the low-lying levels of Pb^{205}.

The low-lying levels of Pb^{205} arise from configurations in which three neutrons are missing from the top neutron shell. The missing neutrons are designated as $P_{1/2}$, $f_{5/2}$, $P_{3/2}$, $i_{13/2}$, and $f_{7/2}$. (For the standard notations of angular momentum states, see any atomic physics book.) With respect to these configurations, Pb^{205} can be considered a "doubly magic core with three neutron holes." ("Core-excited" levels, which arise from configurations in which a nucleon is excited to an orbital in the next shell, will not be considered for this method.)

The energy level of a particular configuration is the amount of energy necessary to form that configuration (by removing neutrons from the stable state). Just as there is a characteristic work function for the removal of an electron from a metal, there is a definite energy required to remove a neutron from the core. This separation energy will be referred to as W.

The Bacher-Goudsmit procedure, in this application, consists of writing an approximate relationship for a particular three-hole energy in terms of corresponding two-hole and one-hole energies, which are known experimentally. To find this relationship, consider first the one-hole states. If A, B, and C each represent a different state, then W(A), W(B), and W(C) will represent their associated energies. Similarly W(AB), W(BC), and W(AC) are the energies associated with two-hole states, and W(ABC) is the energy associated with a three-hole state.

To a zeroth-order approximation, the energy required to remove 2 neutrons simultaneously is equivalent to the sum of the energies required to remove each separately:

$$W_0(AB) = W(A) + W(B) \qquad (1)$$

In a better approximation, a term should be included to account for any interaction between the 2 neutrons. Let this energy be represented by $\omega(AB)$, the pair energy of interaction.

$$W_1(AB) = W(A) + W(B) + \omega(AB) \qquad (2)$$

Similarly, there are energies associated with the other two combinations of A, B, and C:

$$W_1(AC) = W(A) + W(C) + \omega(AC) \qquad (2)$$

$$W_1(BC) = W(B) + W(C) + \omega(BC) \qquad (2)$$

The zeroth-order approximation for three neutrons can be written in the same manner:

$$W_0(ABC) = W(A) + W(B) + W(C) \qquad (3)$$

A higher approximation would also include all the possible pair energies:

$$W_1(ABC) = W(A) + W(B) + W(C) + \omega(AB) + \omega(AC) + \omega(BC) \qquad (4)$$

By solving for the pair energies in equations (a) and substituting into equation (4), the latter can be written (2) without the pair energies:

$$W_1(ABC) = W(AB) + W(AC) + W(BC) - W(A) - W(B) - W(C) \qquad (5)$$

Equation (5) expresses the approximate relationship between the three-hole energy and its components, the two-hole and one-hole energies.

Experimental values for the two-hole and one-hole energies are used in all the calculations. Selected energies are summarized in Table I.

TABLE I
NUCLEAR ENERGY LEVELS

$Pb\ 207$[2]		$Pb\ 206$[3,4]	
Configuration	KeV	Configuration	KeV
$P_{1/2-}$	0	$(P_{1/2}^2)_{0+}$	0
$f_{5/2-}$	569	$(P_{1/2}f_{5/2})_{2+}$	803
$P_{3/2-}$	894	$(P_{1/2}f_{5/2})_{3+}$	1341
$i_{13/2+}$	1633	$(P_{3/2}^2)_{0+}$	2150
$f_{7/2-}$	2338	$(f_{5/2}^2)_{0+}$	1340
		$(P_{1/2}P_{3/2})_{1+}$	1710
		$(P_{1/2}P_{3/2})_{2+}$	1450
		$(P_{1/2}i_{13/2})_{7-}$	2200
		$(P_{1/2}i_{13/2})_{6-}$	2385

In Table I, $(P_{1/2}^2)_{0+}$ has the following meaning: there are two particles in the $P_{1/2}$ sub shell with total angular momentum $I = 0$ and positive parity.

Several of the low-lying states of Pb^{205} have the following estimated configurations:

$$(P_{1/2}{}^2f_{5/2})_{5/2}, \quad (P_{1/2}{}^2P_{3/2})_{3/2}, \text{ and } (P_{1/2}{}^2i_{13/2})_{13/2}.$$

The energies of these three configurations can be readily computed, because each contains a pair of $P_{1/2}$ factors. Assuming that the two $P_{1/2}$ configurations will pair up first (they are in the same orbital), they must pair up to a resultant spin of zero, due to the exclusion principle. Thus the final value of I is determined by the 3*rd* factor.

As an example, the energy of the $(P_{1/2}{}^2f_{5/2})_{5/2}$ configuration will be computed in detail. If this is to be represented by W(ABC), then A is equivalent to $P_{1/2}$, B to $P_{1/2}$, and C to $f_{5/2}$. AB is then fixed as $(P_{1/2}{}^2)_0$, but AC and BC can each have the configuration $(P_{1/2}f_{5/2})_2$ or $(P_{1/2}f_{5/2})_3$, or some combination of the two. Thus weighting factors must be assigned to each, depending on their probability of occurrence.

These weighting factors can be evaluated with respect to a magnetic axis. The $(P_{1/2}f_{5/2})_2$ configuration can take one of $(2I + 1) =$ "five orientations," while the $(P_{1/2}f_{5/2})_3$ configuration can take one of seven. Thus the weights are 5/12 and 7/12, respectively.

These can be substituted into equation (5) to solve for the $(P_{1/2}{}^2f_{5/2})_{5/2}$ configuration energy.

$$W(P_{1/2}{}^2f_{5/2})_{6/2} = W(P_{1/2}{}^2)_0 + 2(5/12)W(P_{1/2}f_{5/2})_2 + 2(7/12)W(P_{1/2}f_{5/2})_3 - W(P_{1/2}) - W(P_{1/2}) - W(f_{5/2})$$

The factor of 2 has been included since the combination occurs twice. This gives a resultant energy of 1665 KeV. This particular configuration has been chosen as the ground state, so 1665 KeV must be subtracted from all other calculations.

This procedure can be applied to two other configurations:

$$(P_{1/2}f_{5/2}{}^2)_{1/2,3/2,5/2,7/2,9/2} \quad \text{and} \quad (P_{1/2}P_{3/2}{}^2)_{1/2,3/2,5/2} .$$

In each of these cases, I can take a whole range of values. For simplicity, only $I = 1/2$ was considered in each case. The calculations can be performed in the same manner as before, since $(f_{5/2}{}^2)_1$ and $(P_{3/2}{}^2)_1$ are not allowable configurations.

The two other possible types of configurations will not be considered. They include those for which all three factors are identical, such as

$$(f_{5/2}{}^3)_{3/2,5/2}$$

and those for which all three differ, such as

$$(P_{1/2}f_{5/2}P_{3/2})_{1/2,3/2,5/2,7/2,9/2} .$$

This method should be valid with proper application of vector addition to the various angular momenta.

The results for the five configurations are summarized in Table II. The $(P_{1/2}{}^2f_{5/2})_{5/2}$ configuration was chosen as the ground state.

Calculations on Pb^{205} have also been made by True and Pryce[5], yielding fair accuracy but at the cost of rather high complexity. As can be seen from Table II, the accuracy of this method is not quite that of True and Pryce. It is quite probable that the accuracy of this method might be improved by making detailed use of the symmetry properties of the wave functions, and also by using improved values for the Pb^{207} and Pb^{208} energies which were used as experimentally determined parameters.

TABLE II

NUCLEAR LEVELS OF Pb^{205}

Configuration	Calculated Energy (KeV)	Experimental Energy (KeV)[6]
$(P_{1/2}{}^2f_{5/2})_{5/2}-$	0	0
$(P_{1/2}{}^2P_{3/2})_{3/2}-$	546	263 (?)
		576
$(P_{1/2}f_{5/2}{}^2)_{1/2}-$	771	
$(P_{1/2}{}^2i_{13/2})_{13/2}+$	1274	1014
$(P_{1/2}P_{3/2}{}^2)_{1/2}-$	1827	

The method should be applicable to other isotopes than Pb^{205}, however the number of neutron holes should be kept small. The removal of a nucleon causes a non-negligible change in the nuclear radius, and in the energy levels of the remaining nucleons.

REFERENCES

1. Bacher and Goudsmit, Phys. Rev. *46*, 948 (1934).
2. D. E. Alburger and A. W. Sunyar, Phys. Rev. *99*, 695 (1955).
3. D. E. Alburger and M. H. L. Pryce, Phys. Rev. *95*, 1482 (1954).
4. W. W. True and K. W. Ford, Phys. Rev. *109*, 1675 (1958).
5. W. W. True, Nuclear Phys. *25*, 155 (1961).
   M. H. L. Pryce, Nuclear Phys. *2*, 226 (1956/57).
6. S. H. Vegors, Jr., R. L. Heath, and P. G. Proctor. (Private Communication.)