STRETCHING A BARBED WIRE FENCE

Perry H. Rahn
Department of Geology & Geological Engineering
South Dakota School of Mines & Technology
Rapid City, SD 57701

Andrew B. Rahn
Adobe Systems, Inc.
Arden Hills, MN 55112

ABSTRACT

A barbed wire suspended between two fence posts forms a catenary. Fixed at one post, a 213 ft span of barbed wire was tightened to 113 lb tension by a fence stretcher located near the other end post. The sag in the middle of the catenary was 3.30 ft. Tightening the wire to 177 lb reduced the sag to 2.30 ft. This single catenary was then lifted onto nails in a straight line on three evenly-spaced in-between posts; thus reducing the tension to 158 lb. Based on the wire weight, a mathematical analysis shows the theoretical cable tension to be reasonably close to the observed value. The barbed wire posses some elasticity, and Young's Modulus was estimated to be 15.8 X 10^6 psi.

Keywords

Catenary, barbed-wire fence

INTRODUCTION

Catenary, a word derived from Latin, means chain. Today the word catenary refers to the shape taken under the influence of gravity by a chain or flexible cable of uniform density freely suspended between two fixed points.

The mathematics of the shape of a catenary, and its application to suspension bridges, goes back hundreds of years. Galileo in 1638 thought the shape of a hanging chain was a parabola, the curve of a projectile in flight (Boyer, 1991). Cables hanging under their own weight are loaded uniformly along the horizontal, and do not form a perfect parabola (Beer and Johnston, 1977). The difference between a parabola and a catenary is small, however, when the cable is tight. In the early part of the eighteenth century Bernoulli formulated complex catenary equations under different loadings, and showed the effects of elasticity ("stretch") of a cable by incorporating Hooke's law into the equations. The mathematics of inelastic and elastic cables, including those used for suspension bridges, is given by Irvine (1981). The dynamics of suspended cables involves rigorous mathematics and has numerous applications ranging from the aerodynamic failure of the Tacoma Narrows bridge in 1940 to the pitch of vibrating stringed instruments as a result of tension.

Figure 1 shows a profile of a uniform inextensible cable hanging from two fixed end points (A and B) at the same elevation. “Span” is the straight-line distance between the end points, and is designated by the letter L. “Sag” is the deflection to the lowest point, and is designated by the letter h. For this paper, using Cartesian coordinates, x and y are the horizontal and vertical distances from the center point C.

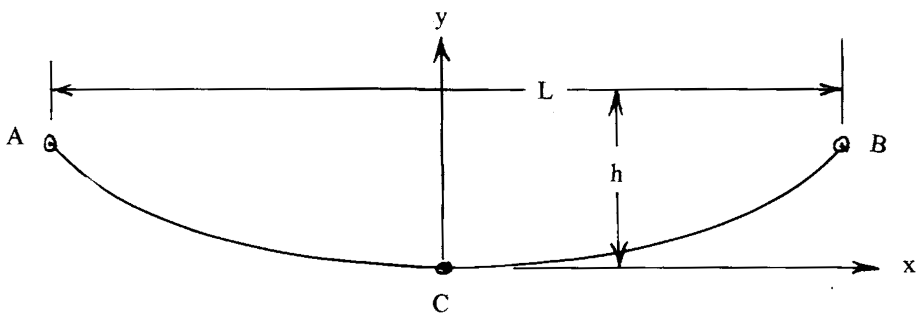

Figure 1. Cartesian coordinates for a cable suspended between two fixed points.

BARBED WIRE FENCE EXPERIMENT

The purpose of this paper is to evaluate the applicability of catenary equations during the installation a barbed wire fence. Barbed wire is a cable and follows the mathematical analyses developed for cables.

In October, 2002, a fence was built near Hill City, SD, on the author’s property. The area is located on a nearly flat flood plain of Slate Creek, and the end posts are very nearly at the same elevation. Figure 2 is a sketch of the fence installation. The span is 213 ft. [Note: for the purpose of the accuracy required in the following calculations, it is assumed that this span measurement is accurate to six significant figures.] The end posts are typical in that they con-

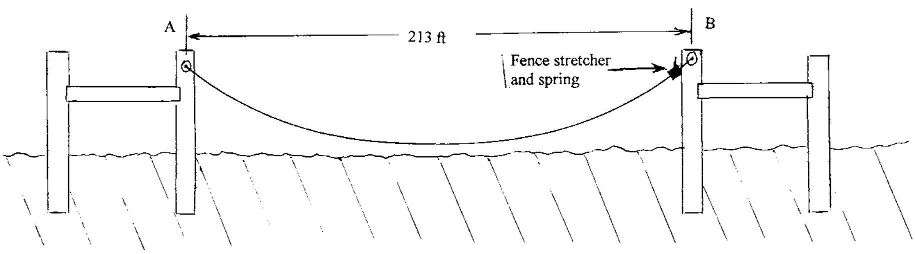

Figure 2. Sketch of barbed wire and fence posts at Slate Creek. The span is 213 ft.

sist of double posts supported by a cross beam and diagonal wires. A barbed wire was to be attached to the end posts approximately 3 ft above the ground. It is assumed that the end posts at A and B are completely immovable and the span is a constant 213 ft. End point A is the location of a staple ("U-nail") firmly attaching the wire. Point B is the location of another staple that has not been firmly affixed; the wire is free to move through it.

The barbed wire is double-wound, galvanized, two point, Sierra 12¾ gauge barbed wire. Each strand has a diameter equal to 1/12.75 = 0.07843 inch = 1.99 mm. A roll of barbed wire 80 rods in length (1320 ft) weighs 68 lb. [Note: in the English system the unit of force is pound (of force). To avoid ambiguity this is often referred to as lb_f. In the metric system the unit of force is a Newton.] The unit weight (w) of the barbed wire equals 68 lb_f/1320 ft = 0.0515 lb_f/ft.

To develop tension in the wire, a standard fence tightener ("fence stretcher") was utilized. This device was located very near end post B (Fig. 3). A calibrated spring was attached to the fence stretcher so that tensional force could be measured. The spring is capable of measuring up to 200 lb force, and was calibrated before and after the experiment to ensure that it functioned correctly.

Figure 4 shows the initial setup when 113 lb of tensional force was applied to the wire. This lifted the wire off the grass, forming a catenary. The deflection at 1/8 spacing between the end points A and B is shown. The sag (in the center) is 3.03 ft. The reason the deflection is not perfectly symmetrical is undoubtedly due to the weight (7 lb) of the fence stretcher located 2 ft from end

Figure 3. Photograph of end post B showing fence stretcher and spring.

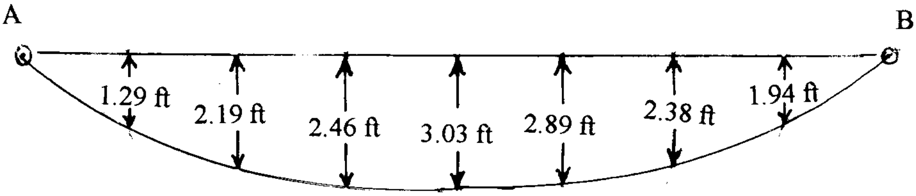

Figure 4. Initial setup using 113 lb tension, showing deflection at 1/8 spacing.

point B. The weight of this device actually constitutes a small point load, but for the purpose of this experiment does not negate the general conclusions relating to a wire forming a catenary by its own unit weight.

The fence stretcher was then cranked tighter, increasing the tension while reducing the sag. At 127 lb the sag was 2.76 ft, at 138 lb the sag was 2.60 ft, at 157 lb the sag was 2.35 ft, and finally at 177 lb the sag was 2.30 ft.

At this last setup (177 lb tension), the wire was then manually lifted and loosely draped over a nail in the exact center of the 213 ft span. This nail is on a straight line between A and B; its coordinates are (0,h). Two catenaries were thus formed, and the tension was observed to drop from 177 lb to 162 lb. The sag in the middle of the two catenaries averaged 0.54 ft.

The wire was then further lifted so as to be draped over nails on a straight line on three posts evenly spaced between end posts A and B. Four catenaries were thus formed, and the tension dropped to 158 lb. The sag in the middle of the four catenaries averaged 0.14 ft.

WIRE LENGTH AND EXTENSION

The theoretical basis for catenary length is shown in Beer and Johnston (1977). If the sag (h) is small relative to the span (L), then the length (s_B) of a suspended cable is:

$$s_B = x_B\{1 + 2/3(y_B/x_B)^2 - 2/5(y_B/x_B)^4 + \ldots\}.$$

Figure 1 shows end point B that has the coordinates x_B and y_B. Because the origin of the Cartesian system is point C, s_B is the cable length from point C to point B only. Since the span is 213 ft, x_B = 106.5 ft. Accordingly, y_B is the sag (h). Using the initial setup where h = 3.03 ft, the cable length is:

$$\begin{aligned} s_B &= 106.5 \text{ ft}\{1 + 2/3(3.03 \text{ ft}/106.5 \text{ ft})^2 - 2/5(3.03 \text{ ft}/106.6 \text{ ft})^4 + \ldots\} \\ &= 106.5 \text{ ft}\{1 + 0.000{,}539{,}6 - 0.000{,}000{,}066 + \ldots\} \\ &= 106.555{,}746 \text{ ft}. \end{aligned}$$

Therefore the entire cable length is 213.114,92 ft, which can be rounded off to 213.115 ft.

Similarly, when the sag was reduced to 2.30 ft, the cable length would be only 213.066 ft. The difference between these two (theoretical) lengths is 0.049 ft.

When the fence stretcher was cranked from 113 to 177 lb, it was observed that 0.3125 ft of barbed wire actually advanced through it, thus diminishing the length of the wire. This advance was partially offset by the attached spring which extended 0.1823 ft as the stretcher was cranked. Therefore the net advance of the wire towards point B was 0.1302 ft.

Considering both the change in the (theoretical) length of the catenary (0.049 ft) during the 113 lb and 177 lb setups, and the net advance of the wire towards point B (0.1302 ft), the barbed wire actually extended 0.1302 ft – 0.049 ft = 0.0812 ft. This shows that the barbed wire is not completely inelastic. [Most theoretical catenary loading equations assume the wire is completely rigid, somewhat like a steel chain. But an elastic material behaves differently. Consider, for example, a very long coiled spring hung as a catenary. If this spring were lifted in the center to form two catenaries the change in length of the spring would be so negligible compared to the total span length that there would be practically no reduction in tension at all.]

Based on the elasticity measurements cited above, a crude determination of Young's modulus can be made (Fig. 5). It is assumed that stress and strain plotted on arithmetic paper form a straight line through the origin (Hooke's Law). Figure 4 shows that the wire extended 0.0812 ft when the tension increased from 113 to 177 lb. This increment is plotted along a straight line going through the origin. The slope of the line can be used to determine Young's modulus (also called E, the Modulus of Elasticity). The slope of this line is 0.0014 ft/lb. Since the wire is 213.115 ft long, the strain is 6.569 X 10^{-6} ft/ft per pound. The inverse of this is a more common way of expressing Hooke's law; therefore this can be restated as 1.52 X 10^5 lb_f are required to produce a strain of 1 ft/ft. In the English system, the units for Young's modulus are pounds (force) divided by square inches (cross sectional area). The cross sectional area of the two wires = 2 (3.1416) (0.995 mm)^2 = 6.2205 mm^2 = 0.9641 X 10^{-2} in^2. Therefore Young's modulus is 1.52 X 10^5 lb_f/0.9641 X 10^{-2} in^2 = 15.8 X 10^6 psi. This is reasonable close to published values of Young's Modulus for steel, approximately 27 to 30 X 10^6 psi (Marks, 1941; Sears and Zemansky, 1955). It is possible that some of the extension observed in this experiment may have been

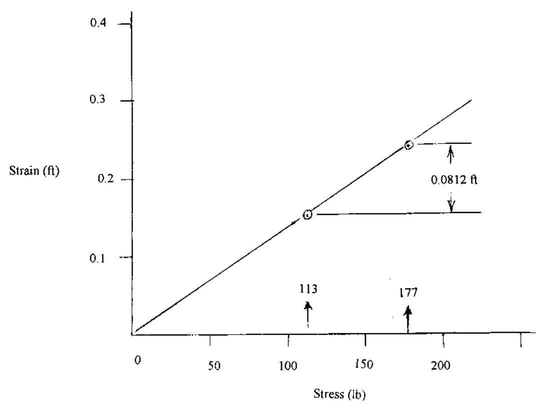

Figure 5. Data used for calculation of Young's modulus. The plot shows the change in length of the 213 ft span based on different tensional loadings.

plastic-like deformation as the barbed wire straightened along kinks as it was stretched.

DEFLECTION AND TENSION

Theoretically, the profile of an inextensible cable hanging from two fixed points at the same elevation forms a catenary. The equation for the form of a catenary utilizes a hyperbolic cosine function (Irvine, 1981). This complex formula is a result of the changing slope of the cable, and the fact that the vertical load per horizontal cable length is not everywhere the same as the vertical load measured along the inclined cable. As pointed out by Beer and Johnston (1977): "…certain catenary problems involve transcendental equations which must be solved by successive approximations." However, where the sag is small relative to the span, a hanging cable can be analyzed as a simpler parabolic curve as follows (after Beer and Johnston, 1977).

Figure 6 shows the forces involved in a catenary. The uniformly distributed load (w) is simulated by a point load (W) so that:

$$W = w\ (\text{cable length}).$$

Using the initial setup:

$$\begin{aligned} W &= 0.0515\ \text{lb}_f/\text{ft}\ (213.115\ \text{ft}) \\ &= 10.9754\ \text{lb}_f. \end{aligned}$$

This vertical load occurs at the middle of the cable (point C on Figure 6A), and this would be supported by $W/2 = 5.4877\ \text{lb}_f$ upward force at the end points A and B.

For the initial setup, the sag was 3.03 ft. Considering the right half of the catenary (from point C to B), half of the total load (W/2) is shown acting at a distance $L/4 = 53.25$ ft from point B. The tension T_C in the cable at point C can be determined by equating the moments about point B:

$$\begin{gathered} M_B \text{ clockwise} = M_B \text{ counterclockwise} \\ h\ (T_C) = L/4\ (W/2) \\ 3.03\ \text{ft}\ (T_C) = 53.25\ \text{ft}\ (5.4877\ \text{lb}_f) \\ T_C = 96.4425\ \text{lb}_f \end{gathered}$$

The cable is horizontal at point C; hence T_C acts horizontally. At point B, the horizontal component must be the same as T_C, but point B also has a vertical force of $5.4877\ \text{lb}_f$. Therefore, the resultant cable tension at point B (T_B) can be solved as:

$$\begin{aligned} T_B &= [T_C^2 + (W/2)^2]^{0.5} \\ &= [(96.4425)^2 + (5.4877)^2]^{0.5} \\ &= 96.60\ \text{lb}_f. \end{aligned}$$

During the initial setup 113 lb tensional force was actually observed at point B. This is 17% greater than the 96.60 lb theoretical tensional force. The weight of the stretcher device undoubtedly caused the observed tension to be greater than the theoretical value.

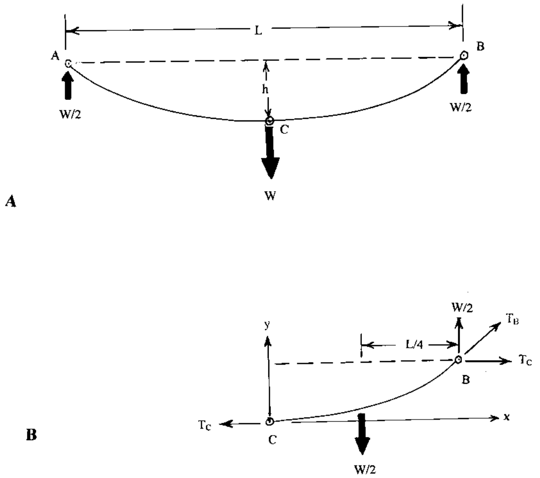

Figure 6. Free-body diagrams for catenary with span (L) and sag (h).

A. Uniformly-distributed load simulated by a point load (W) at the center.

B. Right side of catenary used for balance of moments about point B.

PRACTICAL APPLICATION

When installing a barbed wire fence, there is a natural tendency to hang the wire between the two end posts and hammer it firmly into place. This is the wrong way to do it. A barbed wire, firmly affixed and hanging in a catenary between two fence posts, contains a certain tension. Lifting this wire up onto nails in a straight line on posts in-between the end posts reduces the tension. Therefore, from a practical point of view, the best way to string a barbed wire fence is to first support it in a straight line on as many posts as practical. In other words, lift the wire up into position so that is its draped over a nail (or is free to slide through a staple) on a straight line on all the posts to be utilized between the end posts. Then apply tension to the wire. Then hammer the staples into all the posts. This approach is merely a manifestation of the common dictum that a straight line is the shortest distance between two points.

There are other practical uses of the catenary equations. For example, where a topographic low exists between the two end posts, the barbed wire may initially hang above a post in the valley. If a certain tension is ultimately sought, a sag in this catenary can be precisely established so that the ultimate desired tension is achieved when secured to the central post.

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